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On Fuzzy Λ_γ-Sets and their Applications

Baby Bhattacharya
NIT Agartala, India
Arnab Paul
Techno College of Engineering Agartala, India
and
Jayasree Chakraborty
NIT Agartala, India
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Abstract

The notion of Λ-fuzzy set was introduced by M. E. EI-Shafei and A. Zakari in 2006 [θ-Generalized closed sets in fuzzy topological spaces, The Arabian Journal for Science and Engineering, Vol. 31, Number 2A]. We examine some basic properties of it and prove some characterization theorems for the same. The paper presents a new class of fuzzy sets called fuzzy Λ_γ-sets that includes the class of all fuzzy γ-open sets. It also introduces the notion of fuzzy V_γ-sets as the dual concept of fuzzy Λ_γ sets to study the spaces constituted by those sets and obtain a completely different structure which is called fuzzy independent Alexandorff space. A stronger form of fuzzy Λ_b - continuity [Gulhan Aslim, Gizem Gunel On fuzzy Λ_b -sets and fuzzy Λ_b-continuity. Chaos, Solitons and Fractals 42 (2009). 1024-1030] called fuzzyΛ_γ-continuity is introduced and the relationships are also established with the already existing functions accordingly. Finally, fuzzy Λ_γ-Generalized closed sets are defined and studied with some of their applications.

Keyword: *Fuzzy γ-open set, Fuzzy independent topology, FuzzyΛ_b-set, Fuzzy Λ_γ-set, Fuzzy Λ_b-continuity, Fuzzy Λ_γ-closed set.*

1. Introduction

The potential of the notion of fuzzy set studied by L. A. Zadeh [24] was realized by the researchers and has successfully been applied for new investigations in all the branches of science and technology for more than last five decades. Since Chang [7] defined the concept of a fuzzy topology, then many authors investigated different properties of fuzzy open sets which are weaker than the property of openness of a fuzzy set in a fuzzy topological space. For example, ([19],[3],[25],[21]) have considered such kind of properties of fuzzy sets and most of the collection forms a fuzzy supra topology therein. A significant contribution to the theory of generalized open sets has been reported by A. Csaszar ([8], [9], [10]) and extended by G. Palani Chettry [16] in the context of fuzzy set theory with the name of generalized fuzzy topological space. For example, ([19],[3],[25],[21]) have considered such kind of properties of fuzzy sets and most of the collection forms a fuzzy supra topology therein. A significant contribution to the theory of generalized open sets has been reported by A. Csaszar ([8], [9], [10]) and extended by G. Palani Chettry [16] in the context of fuzzy set theory with the name of generalized fuzzy topological space. Very recently, a different but unique form of fuzzy open sets is being proposed by B. Bhattacharya [6] and established by defining the concept of fuzzy γ-open sets in the sense of D. Andrijevic[1], which are called fuzzy independent open sets as they are totally incomparable with the fuzzy open sets. Also, it is shown that the collection of all fuzzy γ-open sets forms a fuzzy topology itself but does not contain the class of fuzzy open set or not contained in the given fuzzy topology. A finer topology namely Alexandorff topology is established in graphs studied by Jafarian et al. [12] in 2013. It becomes a common approach to study the generalization of topological concepts for a quite long time, for instance ([11], [13]). Our aim is to study the parallel concept of topology in a given fuzzy space with an incomparable nature. More recent works by A. Paul et al. are there in the literature to show different applications of fuzzy γ -open sets ([17], [18]) in the direction of fuzzy bitopology.

In the present paper, we introduce the concept of fuzzy Λ_γ-set (resp. fuzzy V_γ-set) and study some of their properties. We offer a finer fuzzy topology on X than τ_γ-by utilizing the new notion of fuzzy Λ_γ-sets. Finally, we discuss about some fundamental properties of such structure and some related maps. In particular, we have shown that fuzzy Λ_γ -continuity is a stronger form of fuzzyΛ_b-continuity introduce by Aslim et al.[2] in 2009. Lastly, we define fuzzy Λ_γ-generalized closed set and related functions with some comparisons related functions with some comparisons.

As El Nachic [14] contemplated that the concept of fuzzy topology may be relevant to quantum physics in connection with string theory and ϵ^∞-theory, we claim that the results given in the present paper may turn out to be useful in quantum physics.

Let X be any set and I be the closed unit interval [0, 1]. A fuzzy set of X is an element of the set of all functions from X into I. The family of all fuzzy sets in X is denoted by I^X.A fuzzy point x_p [23] is a fuzzy set in X defined by $x_p(x) = p$, $x_p(y) = 0$ for al $x \neq y$, $p \in [0, 1]$ and $x_p \in \mu$ iff $p \leq \mu(x)$.

Throughout this work, by (X, τ) we mean a fuzzy topological space (for short, fts) due to Chang [7] in 1968.The complement of a fuzzy set μ is denoted by μ^C. We consider the following results and definitions for ready references.

1.1. Definition

Let μ be a fuzzy subset of a fts (X, τ) .Then μ is called a fuzzy pre-open set [19] if $\mu \leq int(cl(\mu))$.

1.2. Definition

A subset λ of a fts (X, τ) is said to be fuzzy γ-open set [6] $\lambda \wedge \mu \in FPO(X)$ for each fuzzy pre-open set μ in X. The complement of a fuzzy γ-open is called a fuzzy γ-closed set.

For any fuzzy set δ, $int_\gamma(\delta)$ is the union of all γ-open sets contained in δ in a fts (X, τ). The family of all fuzzy γ-open sets is denoted by FγO(X) and that of fuzzy γ-closed set is denoted by FγC(X).

1.3. Definition

A fuzzy subset μ of a fts (X, τ) is called a Λ-fuzzy set [20] if $\mu = \mu^\Lambda$, where $\mu^\Lambda = \wedge\{\eta : \mu \leq \eta, \eta \in \tau\}$.

1.4. Proposition

[20]

Let (X, τ) be a FT_1 space. Then every fuzzy subset of X is a Λ-fuzzy set.

1.5. Definition

[22]

A fuzzy set η in a fts (X, τ) is called fuzzy dense if there exists no fuzzy closed set μ in (X, τ) such that $\mu > \eta$, that is cl(η)=1_X.

1.6. Definition

[4]

(i) A fuzzy subset λ of a fts (X, τ) is called a generalized fuzzy closed (gfc, for short) fuzzy set if $\lambda \leq \eta$ and $\eta \in \tau$ implies that cl(λ)$\leq \eta$.

(ii) A fts (X, τ) is called fuzzy $T_1/2$ iff every generalized fuzzy closed set in X is fuzzy closed set in X .

1.7. Definition

[5]

A fuzzy set η in a fts (X, τ) is said to be fuzzy locally closed if $\eta = \mu \wedge \beta$, where β is a fuzzy closed set and μ is a fuzzy open set in X .

1.8. Definition

[2]

Let$f : X \rightarrow Y$ be a function from a fts (X, τ) into a fts (Y, σ). Then the function f is called fuzzy Λ_b-continuous if f^{-1} (β) is a fuzzy Λ_b-set of X for each $\beta \in \sigma$.

2. Spome Properties of Λ- Fuzzy Sets

The notion of Λ-fuzzy sets [20] were introduced by M.E. El-Shafei and A. Zakari in 2006. This section isdevoted to study some basic properties of Λ-fuzzy sets to obtain a new kind of fuzzy space. We denote the class of all the Λ-fuzzy sets by FΛ(X) in (X, τ) .

2.1. Lemma

For fuzzy subsets μ, β and $\{\mu_i$:i$\in \Gamma\}$ of a fts (X, τ) the following properties hold:

(i) $\Lambda(0_X)=0_X$ and $\Lambda(1_X)=1_X$.

(ii) $\mu \leq \Lambda(\mu)$.

(iii) $\mu \leq \beta \Rightarrow \Lambda(\mu) \leq \Lambda(\beta)$.

(iv) $\Lambda(\Lambda(\mu)) = \Lambda(\mu)$.
(v) $\Lambda(\vee\mu_i$:i$\in\Gamma$)= $\vee\{\Lambda(\mu_i)$:i$\in\Gamma\}$.
(vi) $\Lambda(\wedge\mu_i$:i$\in\Gamma$) $\leq \wedge\{\Lambda(\mu_i)$:i$\in\Gamma\}$.
(vii) $\Lambda(1_X-\mu)= 1_X-$V(μ).

Proof: We only prove (vi) and (vii). The rest parts can be proved in a similar way.

To prove (vi) Let $\mu=\wedge\{\mu_i$:i$\in\Gamma\}$. Thus, $\mu \leq \mu_i$, for i$\in\Gamma$.
Hence, from (iii)$\Lambda(\mu) \leq \Lambda(\mu_i)$ for all i$\in\Gamma$.
$\Rightarrow \wedge(\wedge\mu_i$:i$\in\Gamma$) $\leq \wedge\{\wedge(\mu_i)$:i$\in\Gamma\}$.
To prove (vii)
1_X-V(μ)=1_X-sup $\{\eta : \eta \leq \mu$, η is a fuzzy closed subset of X}.
=inf $\{\beta$:$1_X-\mu \leq \beta$, β is a fuzzy open subset of X}
=$\wedge(1_X-\mu)$.

2.2. Theorem

For fuzzy subsets μ and { μ_i:i$\in\Gamma$} of a fts (X, τ) the following properties hold:

(i) $\Lambda(\mu)$ is a Λ -fuzzy set.
(ii) If μ is a fuzzy open set, then μ is Λ-fuzzy set.
(iii) If μ_i is a Λ -fuzzy set for each i$\in\Gamma$, then $\vee_{i\in\Gamma}\mu_i$ is a Λ-fuzzy set.
(iv) If μ_i is a Λ fuzzy set for each i$\in\Gamma$, then $\wedge_{i\in\Gamma}\mu_i$ is aΛ -fuzzy set.

Proof: (i) and (ii) are obvious.
(iii) Let $\mu_i\in$FΛ(X) for each i$\in\Gamma$, then by lemma [2.1(vi)], we get
$\vee_{i\in\Gamma}\mu_i= \vee_{i\in\Gamma}\Lambda(\mu_i) = \wedge(\vee_{i\in\Gamma}\mu_i)> \vee_{i\in\Gamma}\mu_i$
So, we have $\vee_{i\in\Gamma}\mu_i = \Lambda(\vee_{i\in\Gamma}\mu_i)$ and
$\vee_{i\in\Gamma}\mu_i\in$F$\Lambda$(X).
(iv) Let $\mu_i\in$FΛ(X) for each i$\in\Gamma$, then by lemma [2.1(vii)], we get
$\wedge_{i\in\Gamma}\mu_i = \wedge_{i\in\Gamma}\Lambda(\mu_i)> \Lambda(\wedge_{i\in\Gamma}\mu_i)>\wedge_{i\in\Gamma}\mu_i$.
Thus, we have $\wedge_{i\in\Gamma}\mu_i = \Lambda$ ($\wedge_{i\in\Gamma}\mu_i$) and $\wedge_{i\in\Gamma}\mu_i$ $\in$FΛ(X).

2.3. Theorem

The class FΛ(X) of all Λ -fuzzy sets forms an Alexandroff topology in a fts (X, τ) and hence (X,FΛ(X)) is a fuzzy Alexandroff space.

Proof: (i) 1_X , 0_X $\in$FΛ(X).

(ii) If μ_i is a Λ-fuzzy set for each i$\in\Gamma$, then $\vee_{i\in\Gamma}\mu_i$ is a Λ -fuzzy set, which is proved in the above theorem.

(iii) If μ_i is a Λ-fuzzy set for each i$\in\Gamma$, then $\wedge_{i\in\Gamma}\mu_i$ is a Λ -fuzzy set,that is proved in the above theorem.

So the pair(X,FΛ(X)) is a fuzzy Alexandroff space.

2.4. Definition

The fuzzy Alexandroff space (X,FΛ(X)) is called the Λ - fuzzy Alexandroff space.

3. Fuzzy Λ_γ -Sets and Related Results

3.1. Definition

Let μ be a fuzzy subset of a fts (X, τ) . Then the fuzzyγ -kernel of μ is denoted by $\Lambda_\gamma(\mu)$ and defined as below:

$\Lambda_\gamma(\mu) = \wedge\{\eta{:}\mu \leq \eta,\ \eta \in$ FγO(X)$\}$.

We define $V_\gamma(\mu)$ $=\vee\{\beta{:}\beta \leq \mu,\ \mu \in$ FγC(X)$\}$.

3.2. Lemma

For any fuzzy subsets μ , β and $\{\ \mu_i$:i$\in \Gamma\}$ of a fts (X, τ) the following properties hold:

(i) $\Lambda_\gamma(0_X)=0_X$ and $\Lambda_\gamma(1_X)=1_X$.

(ii)$\mu \leq \Lambda_\gamma(\mu)$.

(iii) $\mu \leq \beta \Rightarrow \Lambda_\gamma(\mu) \leq \Lambda_\gamma(\beta)$.

(iv)$\Lambda_\gamma(\Lambda_\gamma(\mu)) = \Lambda_\gamma(\mu)$.

(v) If $\mu \in$FγO(X), then $\mu = \Lambda_\gamma(\mu)$.

(vi)$\Lambda_\gamma(\vee\mu_i$:i$\in \Gamma)= \vee\{\Lambda_\gamma(\mu_i)$:i$\in \Gamma\}$.

(vii)$\Lambda_\gamma(\wedge\mu_i$:i$\in \Gamma) \leq \wedge\{\Lambda_\gamma(\mu_i)$:i$\in \Gamma\}$.

(viii) $\Lambda_\gamma(1_X-\mu)= 1_X-V_\gamma(\mu)$.

Proof: The entire proof is similar to that of Lemma 2.1 in section 2 along with the definition of fuzzy Λ_γ -set.

3.3. Definition

Let μ be the fuzzy set of a fts (X, τ). Then μ is called

(i) a fuzzy Λ_γ-set if $\mu = \Lambda_\gamma(\mu)$,

(ii) a fuzzy V_γ-set if $\mu = V_\gamma(\mu)$.

The class of all fuzzy Λ_γ-sets in a fts (X, τ) is denoted by FΛ_γ(X).

Note: One can easily verify that the concepts of Λ -fuzzy set and fuzzy Λ_γ-set are independent of each other, since the notion of open set and γ-open set are completely independent of each other.

3.4. Theorem

For a fts (X, τ), the pair (X, FΛ_γ(X)) is a fuzzy Alexandroff space.
Proof: The proof is straightforward from the above lemma 3.2.

3.5. Remark

The fuzzy Alexandroff spaces obtained in theorem 2.3 of section 2 and Theorem 3.4 of this section respectively, are independent of each other as so is fuzzy open set and fuzzy γ-open set.

3.6. Definition

The space (X, FΛ_γ(X)) is called the fuzzy Λ_γ-Alexandroff space which is termed as fuzzy Independent Alexandroff space.

3.7. Remark

Every fuzzy Λ_γ -set is a fuzzy Λ_b-set, but the converse is not true as every fuzzy b-open set is not a fuzzy γ-open set in general and it follows from the example illustrated below:

3.8. Example

Let $X = \{x, y\}$ and

τ=$\{\{(x, 0.3), (y, 0.6)\}$,$\{(x, 0.2), (y, 0.9)\}$,$\{(x, 0.3), (y, 0.9)\}$,$\{(x, 0.3), (y, 0.6)\}$,0_X,$1_X\}$.

We have FPO(X)=$\{\{(x, p), (y, q)\}$, where p>0.8 or q>0.4$\}$.

Here, $\{(x, 0.1), (y, 0.5)\}$ is a fuzzy Λ_b -set, but it fails to be a fuzzyΛ_γ -set.

3.9. Definition

A fts (X, τ) is called Fγ T_1 space if for any two distinct points x_p, y_q∈X, there exist fuzzy γ neighborhood β and δ in X such that $\beta(y_q)$=0,$\beta(x_p)$>0 and $\delta(x_p)$=0,$\delta(y_q)$>0.

3.10. Remark

Every fuzzy singleton set $\{x_p\}$ in Fγ -T_1 space is a fuzzy γ -closed.

3.11. Theorem

Let (X, τ) be a Fγ -T$_1$ space .Then every fuzzy subset of X is fuzzy Λ_γ -set.

Proof: Let μ is any fuzzy subset in X such that $\mu(x_p)$=0.Now, since X is an Fγ-T$_1$ space, then x_p^c is a fuzzy γ-open set which follow from the above remark. Therefore, $\Lambda_\gamma(\mu)\leq x_p^c \Rightarrow \Lambda_\gamma(\mu(x_P))$=0. Thus $\Lambda_\gamma(\mu) \leq\mu$.Hence the proof.

3.12. Remark

As the fuzzy open set and fuzzy γ -open set are independent to each other in a fts , therefore we can conclude that a Fγ -T$_1$ space is not necessarily a FT$_1$ space and vice versa.

3.13. Definition

Let $f : X \to Y$ be a function from a fts (X, τ) into a fts (Y, σ) . Then the function f is called fuzzy Λ_γ (resp. V$_\gamma$)-continuous if $f^{-1}(\beta)$ is a fuzzy Λ_γ (resp. V$_\gamma$)-set of X for each $\beta \in \sigma$.

3.14. Theorem

Let $f : X \to Y$ be a function from a fts (X, τ) into a fts (Y, σ) .Then the following conditions are equivalent:

(i) $f$is fuzzy Λ_γ -continuous function.
(ii) Inverse image of the fuzzy closed set in Y is a fuzzy V$_\gamma$-set in X.
(iii)f(V$_\gamma(\alpha)$)$\leq$cl$f(\alpha)$,for any fuzzy subset α of X .
(iv)(V$_\gamma(f^{-1}(\delta))\leq f^{-1}$(cl($\delta$)), for any fuzzy subset δ of Y .
(v) (Λ_γ (f^{-1})(δ^C))$\geq f^{-1}$(int(δ^C)).

Proof: (i)$\Rightarrow$(ii) Let η be any open set in Y. Now, since f is a fuzzy Λ_γ -continuous, thus $f^{-1}(\eta)$ is a Λ_γ - set in X. This implies that $1_X - f^{-1}(\eta)$ is a ($1_X - \Lambda_\gamma$)- set in X that is $f^{-1}(\eta^C)$ is a V$_\gamma$ -set in X .

(ii)$\Rightarrow$(iii) From (ii) we get V$_\gamma$(f^{-1}cl(f(A)))=f^{-1}(cl(f(A)).
Now V$_\gamma$(A)$\leq$V$_\gamma$(f^{-1}(f(A))
$\Rightarrow$V$_\gamma$(A)$\leq$V$_\gamma$(cl(f^{-1}(f(A)))
=f^{-1}(cl(f(A)))
Thus, f(V$_\gamma(\alpha)$)$\leq$cl$f(\alpha)$.

(iii)$\Rightarrow$(iv) For any fuzzy subset δ in Y, (iv) can be obtained from (iii).

(iv)⇒(v) If we take complement on both sides of (iv) then (v) can be derived easily.

(v)⇒(i) As the intersection of any two fuzzy Λ_γ -set is fuzzy Λ_γ-set, thus by (v) we can say that f is a fuzzy Λ_γ -continuous.

3.15. Remark

Every fuzzy Λ_γ -continuous function is a fuzzy Λ_b- continuous function. But the converse is not true and that follows from the above remark 3.7.

4. Generalization of Fuzzy Λ- Closed Sets

4.1. Definition

A fuzzy subset λ of a fts (X, τ) is called fuzzy Λ -closed set if $\lambda = \mu \wedge \delta$,where μ is a Λ -fuzzy set and δ is a fuzzy closed set. The family of all fuzzy Λ -closed sets is denoted by F Λ C(X).

A fuzzy subset λ of a fts (X, τ) is called fuzzy Λ-open if its complement is fuzzy Λ-closed set.

4.2. Remark

Every fuzzy locally closed set is a fuzzy Λ -closed set .

4.3. Theorem

Every fuzzy generalized closed set of a fts (X, τ) is a fuzzy closed set iff it is fuzzy Λ-closed set.

Proof: Let λ be a fuzzy generalized closed set. Thus cl($\lambda \leq \mu$), whenever $\lambda \leq \mu$, where $\mu \in \tau$.

Now, since λ is fuzzy Λ-closed set, then $\lambda = \Lambda(\lambda) \wedge \mathrm{cl}(\lambda)$, since $\Lambda(\lambda) \geq \mathrm{cl}(\lambda)$. Therefore, λ is a fuzzy closed set.

Converse part follows directly from the fact that every fuzzy closed set is a fuzzy Λ-closed set.

4.4. Proposition

In a fts (X, τ) ,the following conditions are equivalent:

(i) (X, τ) is fuzzy $T_{1/2}$ space.

(ii) Every fuzzy generalized closed set is a fuzzy Λ- closed set.

Proof: (i)$\Rightarrow$(ii) Given (X, τ) be a fuzzy $T_{1/2}$ space.Hence, from the definition we have that every fuzzy generalized closed set is fuzzy closed set and it is a fuzzy Λ-closed set.

Proof: (ii)$\Rightarrow$(i) Using theorem 3.14, it can be proved easily.

4.5. Definition

A fuzzy subset λ of a fts (X, τ) is called fuzzy Λ_γ-closed set if $\lambda = \mu \wedge \delta$,where μ is a fuzzy Λ_γ-set and δ is an fuzzy closed set. The family of all fuzzy Λ_γ -closed sets is denoted by $F\Lambda_\gamma C(X)$.

The fuzzy subset λ of the fts (X, τ) is called a fuzzy Λ_γ-open if its complement is fuzzy Λ_γ- closed set.

4.6. Lemma

For a fuzzy subset λ of a fts (X, τ), the following conditions are equivalent :

(i) λ is a fuzzy Λ_γ -closed set.
(ii) $\lambda = \mu \wedge cl(\lambda)$, where μ is a fuzzy Λ_γ-set .
(iii)$\lambda = \Lambda_\gamma(\lambda) \wedge cl(\lambda)$.

Proof: (i)$\Rightarrow$(ii) Let λ be a fuzzy Λ_γ-closed set.

Therefore, $\lambda = \mu \wedge \delta$, where μ is a fuzzy Λ_γ-set and δ is an fuzzy closed set. Since $\lambda \leq \delta$ implies $cl(\lambda) \leq \delta$ and

$\lambda = \mu \wedge \delta \geq \mu \wedge cl(\lambda) \geq \lambda$.

Therefore, we have $\lambda = \mu \wedge cl(\lambda)$.

(ii)$\Rightarrow$(iii) Let $\lambda = \mu \wedge cl(\lambda)$,where μ is a fuzzy Λ_γ-set.

Since $\lambda \leq \mu$ and it implies that

$\Lambda_\gamma(\lambda) \leq \mu$ and $\lambda = \mu \wedge \delta \geq \Lambda_\gamma(\lambda) \wedge cl(\lambda) \geq \lambda$.

Therefore, we have $\lambda = \Lambda_\gamma(\lambda) \wedge cl(\lambda)$.

(iii)$\Rightarrow$(i) It is obvious and hence omitted.

4.7. Remark

Fuzzy Λ_γ -closed set and fuzzy Λ-closed set are independent of each other , since every fuzzy Λ_γ -set and Λ-fuzzy set are independent.

4.8. Proposition

If λ be any fuzzy dense set and fuzzy Λ_γ-closed set, then λ is fuzzy Λ_γ -set.

Proof: Let Λ be any fuzzy dense set and fuzzy Λ_γ closed set, then $\Lambda = \Lambda_\gamma(\lambda) \wedge \text{cl}(\lambda) = \Lambda_\gamma(\lambda) \wedge 1_X = \Lambda_\gamma(\lambda)$. Thus, λ is a fuzzy Λ_γ -set.

4.9. Definition

A fuzzy subset λ in a fts (X, τ) is said to be fuzzy semiγ-closed set if $\text{int}_\gamma(\text{cl}(\lambda)) \leq \lambda$.

4.10. Proposition

A fuzzy Λ_γ-closed set is fuzzy closed set if it is fuzzy semi γ- closed set.

Proof: Let λ be any fuzzy Λ_γ -closed set, so $\lambda = \Lambda_\gamma(\lambda) \wedge \text{cl}(\lambda)$. Given that λ is a fuzzy semi γ - closed set. Thus $\text{int}_\gamma(\text{cl}(\lambda)) \leq \lambda$.

Let $\text{int}_\gamma(\text{cl}(\lambda)) \leq \lambda \leq \delta_k$, k$\in$I where δ_k are fuzzy γ-open set. Therefore, cl (λ) $\leq \wedge\delta_k$.

This implies that cl (λ)$\leq \Lambda_\gamma(\lambda)$.

Therefore, λ =cl (λ) and hence λ is a fuzzy closed set.

4.11. Definition

The fuzzy set λ of a fts (X, τ) is called a fuzzy Λ_γ -generalized closed set if cl (λ)$\leq \mu$ whenever($\lambda \leq \mu$ and μ is fuzzy Λ_γ -open set.

4.12. Proposition

In a fts (X, τ) the union of any two fuzzy Λ_γ -generalized closed sets is again a fuzzym Λ_γ -generalized closed set.

Proof: Let α and β be any two fuzzy Λ_γ-generalized closed sets in X. Thus cl (α) $\leq \mu$,whenever $\alpha \leq \mu$ and cl (β) $\leq \mu$, whenever $\beta \leq \mu$, where μ is fuzzy Λ_γ-open set .

Now, if $\alpha \vee \beta \leq \mu$, then $\text{cl}(\alpha \vee \beta) = \text{cl}(\alpha) \vee \text{cl}(\beta) \leq \mu$. Hence the proof is done.

4.13. remark

In a fts (X, τ),the intersection of any two fuzzy Λ_γ -generalized closed sets may not be a fuzzy Λ_γ generalized closed set as seen in the following example.

4.14. example

Let $X = \{x, y\}$ and

τ={$\{(x, 0.8), (y, 0.3)\}$,0_X,1_X}.

We have FγO(X)={$\{(x, p), (y, q)\}$, where q>0.3 for any value of p}.

Here, $\{(x, 0.6), (y, 0.2)\}$ and $\{(x, 0.3), (y, 0.25)\}$ are two fuzzy Λ_γ -generalized closed sets, but their intersection is not a fuzzy Λ_γ -generalized closed set.

4.15. Definition

Let $f : X \to Y$ be a function from a fts (X, τ) into a fts (y, σ) .Then the function f is called a fuzzyΛ_γ -generalized continuous, if $f^{-1}(\mu)$ is fuzzy Λ_γ -generalized closed in X for each fuzzy closed μ in Y.

4.16. Proposition

Every fuzzy closed set is a fuzzy Λ_γ -generalized closed set in a fts (X, τ).

Proof: Let β be any fuzzy closed set and μ be any fuzzy Λ_γ-open set such that $\beta \leq \mu$, cl(β)=$\beta \leq \mu$.Thus β is a fuzzy Λ_γ-generalized closed set.

4.17. Remark

It is clear from the above example 4.14 that in a fts (X, τ) , every fuzzy Λ_γ -generalized closed set is not a fuzzy closed set .

4.18. Remark

Every fuzzy Λ_γ-closed set need not be a fuzzy Λ_γ-generalized closed set as verified in the following example.

4.19. example

Let $X = \{x, y\}$ and

τ={$\{(x, 0.2), (y, 0.2)\}$,0_X,1_X}.

Here FγO(X)={$\{(x, p), (y, q)\}$, where 0≤p≤0.2 and p>0.8}.

Let us suppose that

$\lambda = \{(x, 0.85), (y, 0.85)\}$ and we have Λ_γ (λ)=λ which implies that λ is a fuzzy Λ_γ-closed set. Again, $\lambda \leq \lambda$ and cl(λ)=$1_X\lambda$. Therefore, λ is not fuzzyΛ_γ- generalized closed set.

4.20. remark

Using the above example, we see that the fuzzy set$\{(x, 0.3), (y, 0.25)\}$ is a fuzzy Λ_γ-generalized closed set but not a fuzzy Λ_γ -closed set .

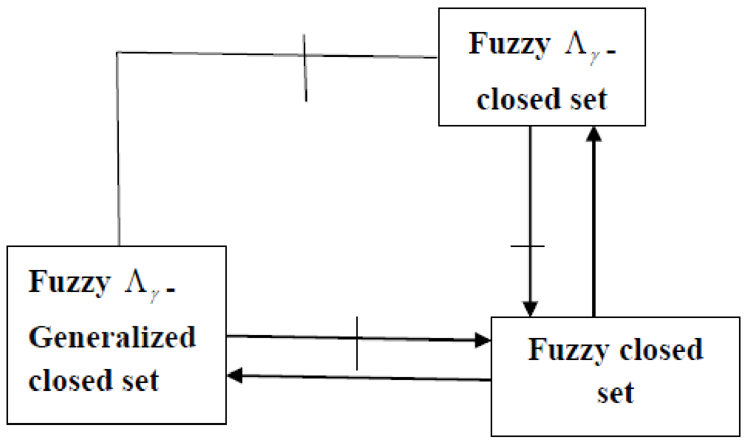
5. Applications

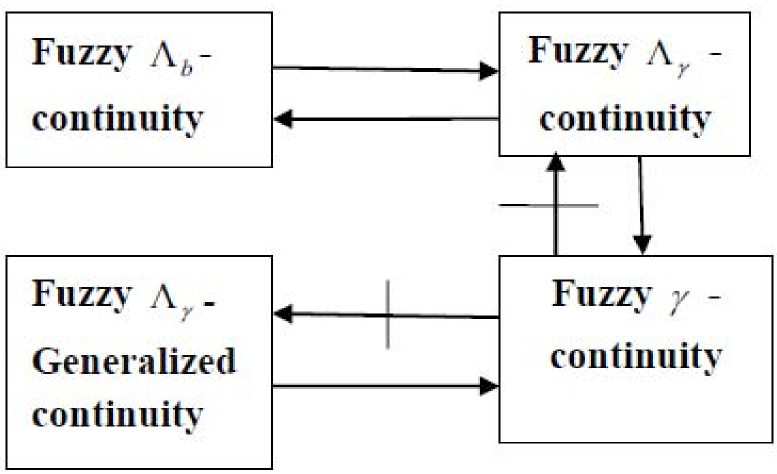
i) B. Bhattacharya has shown that the collection of all γ-open sets forms an independent topology in a fts. But in the present paper, we show that the collection of all Λ_γ -sets forms a stronger independent topology, called fuzzy Λ_γ-Alexandroff space. Thus, this structure may be used for further work on fuzzy independent topology and some related areas.

ii) Also the present study may have application in computational fuzzy topology or in quantum Physics, particularly in connection with string theory and ϵ^∞ theory, etc. and many more as fuzzy topology may be relevant to those concepts .

iii) Generalized closed sets play an important role in the study of morphology. We contemplate that the generalized forms of fuzzy γ-closed sets may be relevant in the study of grey-scale connected operator and inf-structuring function.

From the above discussion, we can draw the following figures in a fuzzy topological space





6. Conclusion

Undoubtedly the worth of fuzzy topology has rapidly been appeared in many fields of both pure and applied sciences where fuzzy open set and fuzzy continuity are playing the key role in the structure of fuzzy topology and as a consequence, the weaker form of fuzzy continuity and open sets have been studied by many researchers. This paper presents a parallel form of fuzzy topology and it is a mathematical existence of parallel topology rather for parallel circuits. Frontier may be done as an application of fuzzy contra γ -continuity in the near future.

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Baby Bhattacharya
Department of Mathematics,
National Institute of Technology,
Agartala, 799046,
India
e-mail : babybhatt75@gmail.com

Arnab Paul
Science and Humanities Department,
Techno College of Engineering Agartala,
Agartala, 799004,
India
e-mail : mrarnabpaul87@gmail.com

and

Jayasree Chakraborty
Department of Mathematics,
National Institute of Technology,
Agartala, 799046,
India
e-mail : chakrabortyjaysree1@gmail.com