

Corrigendum

Corrigendum to “An Extension of Sheffer Polynomials”

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We regret to announce that there are some mathematical errors in theorem 2.3 and theorem 2.4. Our aim to correct and modify the theorem 2.3 and theorem 2.4.

Brown [1] stated that $\{B_n(x)\}$ is a polynomial sequence which is simple and of degree precisely n . $\{B_n(x)\}$ is a binomial sequence if

$$B_n(x+y) = \sum_{k=0}^n \binom{n}{k} B_{n-k}(x) B_k(y) \quad n = 0, 1, 2, \dots$$

and a simple polynomial sequence $\{P_n(x)\}$ is a Sheffer sequence if there is a binomial sequence $\{B_n(x)\}$ such that

$$P_n(x+y) = \sum_{k=0}^n \binom{n}{k} B_{n-k}(x) P_k(y) \quad n = 0, 1, 2, \dots$$

Theorem 2.3: Let $p_n(x, y)$ be symmetric, a class of polynomials in two variables and Sheffer A-type zero which belong to the operator $J(D)$ and have the generating function as equation (1.11) of theorem 2.1 (See [2]). There exist sequences $\alpha_k^{(s)}$, μ_k and η_k , independent of x, y and n , such that for all $n \geq 1$,

$$np_n(x, y) = \sum_{k=0}^{n-1} \sum_{i=1}^r \left(\alpha_k^{(i)} + x\varepsilon_i^{k+1}\mu_k + y\varepsilon_i^{k+1}\eta_k \right) p_{n-k-1}(x, y), \quad (1.15)$$

where $\mu_k = (k+1)g_k$ in terms of g_k of equation (1.8) and $\eta_k = (k+1)h_k$, in terms of h_k of equation (1.9).

Proof: Let (See [2])

$$\begin{aligned}
\sum_{n=0}^{\infty} n p_n(x, y) t^n &= t \left[\sum_{i=1}^r \{A'_i(t) + x \varepsilon_i G'(\varepsilon_i t) A_i(t) + y \varepsilon_i H'(\varepsilon_i t) A_i(t)\} \right. \\
&\quad \left. \exp(xG(\varepsilon_i t)) \exp(yH(\varepsilon_i t)) \right] \\
&= t \left[\sum_{i=1}^r \left\{ \frac{A'_i(t)}{A_i(t)} + x \varepsilon_i G'(\varepsilon_i t) + y \varepsilon_i H'(\varepsilon_i t) \right\} \right. \\
&\quad \left. A_i(t) \exp(xG(\varepsilon_i t)) \exp(yH(\varepsilon_i t)) \right] \\
&= \sum_{n=1}^{\infty} \sum_{k=0}^{n-1} \sum_{i=1}^r \left(\alpha_k^{(i)} + x \varepsilon_i^{k+1} \mu_k + y \varepsilon_i^{k+1} \eta_k \right) p_{n-k-1}(x, y) t^n.
\end{aligned}$$

Thus we get

$$n p_n(x, y) = \sum_{k=0}^{n-1} \sum_{i=1}^r \left(\alpha_k^{(i)} + x \varepsilon_i^{k+1} \mu_k + y \varepsilon_i^{k+1} \eta_k \right) p_{n-k-1}(x, y).$$

This gives the proof of statement.

Theorem 2.4: A necessary and sufficient condition that $p_n(x, y)$ be of Sheffer A-type zero, there exists sequence g_k and h_k , independent of x, y and n , such that

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} \right) p_n(x, y) = \sum_{k=0}^{n-1} \sum_{i=1}^r (\varepsilon_i^{k+1} g_k + \varepsilon_i^{k+1} h_k) p_{n-k-1}(x, y),$$

where $p_n(x, y)$ is symmetric and a class of polynomials in two variables.

Proof: Let (See [2])

$$\sum_{n=0}^{\infty} \left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} \right) p_n(x, y) t^n = \sum_{i=1}^r \{G(\varepsilon_i t) + H(\varepsilon_i t)\} A_i(t) \exp(xG(\varepsilon_i t)) \exp(yH(\varepsilon_i t))$$

$$\begin{aligned}
&= \sum_{i=1}^r \sum_{k=0}^{\infty} (\varepsilon_i^{k+1} g_k + \varepsilon_i^{k+1} h_k) t^{k+1} \sum_{n=0}^{\infty} p_n(x, y) t^n, \\
&= \sum_{n=1}^{\infty} \sum_{k=0}^{n-1} \sum_{i=1}^r (\varepsilon_i^{k+1} g_k + \varepsilon_i^{k+1} h_k) p_{n-k-1}(x, y) t^n.
\end{aligned}$$

Thus

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} \right) p_n(x, y) = \sum_{k=0}^{n-1} \sum_{i=1}^r (\varepsilon_i^{k+1} g_k + \varepsilon_i^{k+1} h_k) p_{n-k-1}(x, y).$$

This is the proof of theorem 2.4.

References:

1. J. W. Brown, On multivariable Sheffer sequences, J. Math. Anal. Appl., Vol. 69, (1979), pp.398-410.
2. A. K. Shukla and S. J. Rapeli, An Extension of Sheffer Polynomials, Proyecciones Journal of Mathematics, Vol. 30, No. 2 (2011), pp. 265-275.