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Six dimensional matrix summability of triple sequences

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Abstract

In this paper we introduced the RH-regularity condition of six dimensional matrix. Matrix summability is one of the important tool used to characterize sequence spaces. In 2004 Patterson presented such a characterization of bounded double sequence using four dimensional matrix. Our main aim is to extend Patterson result in triple sequence spaces using six dimensional matrix transformations.

Key Words: *Triple sequence, RH-regular, regular matrix transformation.*

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1. Introduction and Preliminaries

Throughout the article a triple sequence x is denoted by $(x_{p,q,r})$ i.e. a triple infinite array of real or complex numbers $x_{p,q,r}; p; q; r \in N$. Throughout N , R and C denote the set of natural, real and complex numbers respectively.

At the initial stage different types of notions of triple sequences were introduced and investigated by Sahiner, Gurdal and Duden [3]. Sahiner and Tripathy [4] studied I-related properties in triple sequence spaces. Debnath, Sharma and Das [15] and Debnath and Das [16] generalized these concepts by using the difference operator. Recently Debnath, Das, Bhattacharya and Debnath [17] studied regular matrix transformation on triple sequence spaces and showed some interesting results. Triple sequences have also been studied by Tripathy and Goswami ([5],[6],[7],[8]) and many others.

The Silverman-Toeplitz [13] theorem characterizes the regularity of two dimensional matrix transformation. Robison [10] has given definitions for giving a value to a divergent double series by considering the double sequence for the series and established the conditions of regularity of linear transformations on double sequence spaces. Since then this concept has been studied by many researchers. In 2004 Patterson [14] presented an accessible multidimensional analog of theorem of Brudno [1] using four dimensional matrix transformation of double sequences. In this paper the aim of the author is to extend results of Patterson in triple sequence spaces using six dimensional matrix transformations. In addition, we introduced the RH -regularity condition of six dimensional matrix.

Definition 1.1. A triple sequence $(x_{p,q,r})$ is said to be convergent to L in Pringsheim's sense (denoted by $P - \lim x = L$) if for every $\epsilon > 0$, there exists $n_0 \in N$ such that

$$|x_{p,q,r} - L| < \epsilon, \text{ whenever } p \geq n_0, q \geq n_0, r \geq n_0,$$

and we write

$$\lim_{p, q, r \rightarrow \infty} x_{p,q,r} = L.$$

Note: A triple sequence convergent in Pringsheim's sense is not necessarily bounded [3].

Definition 1.2. A triple sequence $(x_{p,q,r})$ is called definite divergent, if

for every (arbitrarily large) $G > 0$ there exist $n_1, n_2, n_3 \in N$ such that $|x_{p,q,r}| > G$ for $p \geq n_1, q \geq n_2, r \geq n_3$.

Definition 1.3. A triple sequence $(x_{p,q,r})$ is divergent in the Pringsheim sense (P -divergent) provided that $(x_{p,q,r})$ does not converge in the Pringsheim sense (P -convergent).

Definition 1.4. A triple sequence $(x_{p,q,r})$ is said to be bounded if there exist $M > 0$ such that $|x_{p,q,r}| < M$ for all $p, q, r \in N$.

Definition 1.5. Let A denote a six dimensional summability method that maps the complex triple sequence x into the triple sequence Ax where the lmn -th term to Ax is as follows:

$$(Ax)_{l,m,n} = \sum_{p=1}^{\infty} \sum_{q=1}^{\infty} \sum_{r=1}^{\infty} a_{l,m,n,p,q,r} x_{p,q,r}$$

Definition 1.6. The six dimensional matrix A is said to be RH -regular if it maps every bounded P -convergent sequence into a P -convergent sequence with the same P -limit.

Theorem 1.1. ([11],[10]) The four dimensional matrix A is RH -regular if and only if

$$RH_1 : P - \lim_{m,n \rightarrow \infty} a_{m,n,p,q} = 0, \text{ for each } p \text{ and } q ;$$

$$RH_2 : P - \lim_{l,m \rightarrow \infty} \sum_{p=1, q=1}^{\infty, \infty} a_{l,m,p,q} = 1;$$

$$RH_3 : P - \lim_{l,m \rightarrow \infty} \sum_{p=1}^{\infty} |a_{l,m,p,q}| = 0, \text{ for each } q ;$$

$$RH_4 : P - \lim_{l,m \rightarrow \infty} \sum_{q=1}^{\infty} |a_{l,m,p,q}| = 0, \text{ for each } p ;$$

$$RH_5 : \sum_{p=1, q=1}^{\infty, \infty} |a_{l,m,p,q}| \text{ is } P\text{-convergent; and}$$

$$RH_6 : \text{there exist finite positive integers } A \text{ and } B \text{ such that } \sum_{p,q > B} |a_{l,m,p,q}| < A;$$

we introduce the regularity condition of six dimensional matrix as follows:

Theorem 1.2. The six dimensional matrix A is RH -regular if and only if

$RH_1 : P - \lim_{l,m,n \rightarrow \infty} a_{l,m,n,p,q,r} = 0$, for each p, q and r ;

$RH_2 : P - \lim_{l,m,n \rightarrow \infty} \sum_{p=1,q=1,r=1}^{\infty,\infty,\infty} a_{l,m,n,p,q,r} = 1$;

$RH_3 : P - \lim_{l,m,n \rightarrow \infty} \sum_{p=1}^{\infty} |a_{l,m,n,p,q,r}| = 0$, for each q and r ;

$RH_4 : P - \lim_{l,m,n \rightarrow \infty} \sum_{q=1}^{\infty} |a_{l,m,n,p,q,r}| = 0$, for each p and r ;

$RH_5 : P - \lim_{l,m,n \rightarrow \infty} \sum_{r=1}^{\infty} |a_{l,m,n,p,q,r}| = 0$, for each p and q ;

$RH_6 : \sum_{p=1,q=1,r=1}^{\infty,\infty,\infty} |a_{l,m,n,p,q,r}|$ is P -convergent;

$RH_7 : there exist finit positive integers A and B such that $\sum_{p,q,r > B} |a_{l,m,n,p,q,r}| < A$;$

2. Main Results

Theorem 2.1. If $A = (a_{l,m,n,p,q,r})$ and $B = (b_{l,m,n,p,q,r})$ be two six dimensional RH -regular summability matrices sum a bounded triple sequence $(x_{p,q,r})$ to the same sum, then there exist a bounded triple sequence which is summed by B as by A .

Proof. We consider a sequence $(x_{l,m,n})$ such that

$$P - \lim_{l,m,n} (Ax)_{l,m,n} = 0 \text{ and } P - \lim_{l,m,n} (Bx)_{l,m,n} = 0$$

Let

$$\chi_{l,m,n} = \sum_{p=1,q=1,r=1}^{\infty,\infty,\infty} a_{l,m,n,p,q,r} x_{p,q,r} \text{ and } \psi_{l,m,n} = \sum_{p=1,q=1,r=1}^{\infty,\infty,\infty} b_{l,m,n,p,q,r} x_{p,q,r}$$

Since $A = (a_{l,m,n,p,q,r})$ and $B = (b_{l,m,n,p,q,r})$ be two six dimensional RH -regular summability matrices and both are summed to 0, then there exist $\epsilon_{l,m,n} > 0$ with Pringshiem limit zero such that

$$|\chi_{l,m,n}| \leq \epsilon_{l,m,n} \text{ and } |\psi_{l,m,n}| \leq \epsilon_{l,m,n}$$

$$\text{Let } U := \max_{p,q,r} |x_{p,q,r}| \text{ and } V := \max_{p,q,r} |a_{l,m,n,p,q,r}| |x_{p,q,r}|$$

Let the triple partial sum $\sum_{i=1, j=1, k=1}^{p, q, r} s_{i, j, k}$ is denoted by $(H_{p, q, r})$.

Now we have $(H_{p, q, r})$ has the following eight properties

- (i) $|H_{p, p, p}| \leq 1$
- (ii) $|H_{p, p, p} - H_{p, p, p+1}| = 0$
- (iii) $|H_{p, p, p} - H_{p, p+1, p+1}| = 0$
- (iv) $|H_{p, p, p} - H_{p+1, p, p}| = 0$
- (v) $|H_{p, p, p} - H_{p+1, p+1, p}| = 0$
- (vi) $|H_{p, p, p} - H_{p+1, p, p}| = 0$
- (vii) $|H_{p, p, p} - H_{p+1, p, p+1}| = 0$
- (viii) $|H_{p, p, p} - H_{p+1, p+1, p+1}| = \lambda_{p, p, p}$ with $P - \lim_{p, p, p} \lambda_{p, p, p} = 0$

By the RH-regularity conditions of A and B we can choose the following index sequences

$$(l_t), (m_u), (n_v), (\alpha_l), (\bar{\alpha}_l), (\beta_m), (\bar{\beta}_m), (\gamma_n), (\bar{\gamma}_n) \text{ and } (\delta_{l, m, n})$$

Choose $(\alpha_l), (\bar{\alpha}_l), (\beta_m), (\bar{\beta}_m), (\gamma_n), (\bar{\gamma}_n)$ are strictly increasing and $P - \lim_{l, m, n} \delta_{l, m, n} = 0$

By the RH-regularity condition RH_1 gives us

$$\sum_{p=1, q=1, r=1}^{\alpha_l-1, \beta_m-1, \gamma_n-1} |a_{l, m, n, p, q, r}| < \delta_{l, m, n},$$

and

$$\sum_{p=1, q=1, r=1}^{\alpha_l-1, \beta_m-1, \gamma_n-1} |b_{l, m, n, p, q, r}| < \delta_{l, m, n},$$

Now RH_3, RH_4 and RH_5 gives us the following:

$$\sum_{p=1, q=\beta_m, r=\bar{\gamma}_n}^{\alpha_l-1, \bar{\beta}_m-1, \infty} |a_{l,m,n,p,q,r}| < \delta_{l,m,n},$$

$$\sum_{p=1, q=\bar{\beta}_m, r=\gamma_n}^{\alpha_l-1, \infty, \bar{\gamma}_n-1} |a_{l,m,n,p,q,r}| < \delta_{l,m,n},$$

$$\sum_{p=\alpha_l, q=1, r=\bar{\gamma}_n}^{\bar{\alpha}_l-1, \beta_m-1, \infty} |a_{l,m,n,p,q,r}| < \delta_{l,m,n},$$

$$\sum_{p=\bar{\alpha}_l, q=1, r=\gamma_n}^{\infty, \beta_m-1, \bar{\gamma}_n-1} |a_{l,m,n,p,q,r}| < \delta_{l,m,n},$$

$$\sum_{p=\bar{\alpha}_l, q=\bar{\beta}_m, r=1}^{\infty, \bar{\beta}_m-1, \gamma_n-1} |a_{l,m,n,p,q,r}| < \delta_{l,m,n},$$

$$\sum_{p=\alpha_l, q=\bar{\beta}_m, r=1}^{\bar{\alpha}_l-1, \infty, \gamma_n-1} |a_{l,m,n,p,q,r}| < \delta_{l,m,n},$$

and

$$\sum_{p=1, q=\beta_m, r=\bar{\gamma}_n}^{\alpha_l-1, \bar{\beta}_m-1, \infty} |b_{l,m,n,p,q,r}| < \delta_{l,m,n},$$

$$\sum_{p=1, q=\bar{\beta}_m, r=\gamma_n}^{\alpha_l-1, \infty, \bar{\gamma}_n-1} |b_{l,m,n,p,q,r}| < \delta_{l,m,n},$$

$$\sum_{p=\alpha_l, q=1, r=\bar{\gamma}_n}^{\bar{\alpha}_l-1, \beta_m-1, \infty} |b_{l,m,n,p,q,r}| < \delta_{l,m,n},$$

$$\sum_{p=\bar{\alpha}_l, q=1, r=\gamma_n}^{\infty, \beta_m-1, \bar{\gamma}_n-1} |b_{l,m,n,p,q,r}| < \delta_{l,m,n},$$

$$\sum_{p=\bar{\alpha}_l, q=\bar{\beta}_m, r=1}^{\infty, \bar{\beta}_m-1, \gamma_n-1} |b_{l,m,n,p,q,r}| < \delta_{l,m,n},$$

$$\sum_{p=\alpha_l, q=\bar{\beta}_m, r=1}^{\bar{\alpha}_l-1, \infty, \gamma_n-1} |b_{l,m,n,p,q,r}| < \delta_{l,m,n},$$

and RH_6 gives us

$$\sum_{p=\bar{\alpha}_l-1, q=\bar{\beta}_m-1, r=\bar{\gamma}_n-1}^{\infty, \infty, \infty} |a_{l,m,n,p,q,r}| < \delta_{l,m,n},$$

and

$$\sum_{p=\bar{\alpha}_l-1, q=\bar{\beta}_m-1, r=\bar{\gamma}_n-1}^{\infty, \infty, \infty} |b_{l,m,n,p,q,r}| < \delta_{l,m,n},$$

We choose the index sequences (α_{l_t}) , $(\bar{\alpha}_{l_t})$, (β_{m_u}) , $(\bar{\beta}_{m_u})$, (γ_{n_v}) and $(\bar{\gamma}_{n_v})$ such that

$$\alpha_{l_t} = \bar{\alpha}_{l_{t-1}}, \beta_{m_u} = \bar{\beta}_{m_{u-1}} \text{ and } \gamma_{n_v} = \bar{\gamma}_{n_{v-1}} \text{ for all } t, u, v = 1, 2, 3, 4, \dots$$

We consider the following triple sequence

$$\dot{x}_{p,q,r} = \begin{cases} x_{p,q,r} & \text{for } p < \bar{\alpha}_{l_1}, \quad q < \bar{\beta}_{m_1} \quad \text{and } r < \bar{\gamma}_{n_1}, \\ H_{t,u,v}x_{p,q,r} & \text{for } \alpha_{l_t} \leq p \leq \bar{\alpha}_{l_t} \quad \beta_{m_u} \leq q \leq \bar{\beta}_{m_u} \quad \text{and } \gamma_{n_v} \leq r \leq \bar{\gamma}_{n_v}, \end{cases}$$

(2.1)

Now for the inequalities $l_t \leq l < l_{t+1}$, $m_u \leq m < m_{u+1}$ and $n_v \leq n < n_{v+1}$ the above triple sequence $(\dot{x}_{p,q,r})$ gives us the following:

$$\begin{aligned} \dot{\chi}_{l,m,n} &= \sum_{p=1,q=1,r=1}^{\infty,\infty,\infty} a_{l,m,n,p,q,r} \dot{x}_{p,q,r} \\ &= \sum_{p=1,q=1,r=1}^{\alpha_{l_1}-1,\beta_{m_1}-1,\gamma_{n_1}-1} a_{l,m,n,p,q,r} \dot{x}_{p,q,r} + \sum_{1 \leq p < \alpha_{l_1}-1, \beta_{m_1} \leq q \leq \bar{\beta}_{m_1}-1, \bar{\gamma}_{n_1} \leq r \leq \infty} a_{l,m,n,p,q,r} \dot{x}_{p,q,r} \\ &+ \sum_{1 \leq p < \alpha_{l_1}-1, \bar{\beta}_{m_1} \leq q \leq \infty, \gamma_{n_1} \leq r \leq \bar{\gamma}_{n_1}-1} a_{l,m,n,p,q,r} \dot{x}_{p,q,r} + \sum_{\alpha_{l_1} \leq p < \bar{\alpha}_{l_1}-1, 1 \leq q \leq \beta_{m_1}-1, \bar{\gamma}_{n_1} \leq r \leq \infty} a_{l,m,n,p,q,r} \dot{x}_{p,q,r} \\ &+ \sum_{\bar{\alpha}_{l_1} \leq p < \infty, 1 \leq q \leq \beta_{m_1}-1, \gamma_{n_1} \leq r \leq \bar{\gamma}_{n_1}-1} a_{l,m,n,p,q,r} \dot{x}_{p,q,r} + \sum_{\bar{\alpha}_{l_1} \leq p < \infty, \beta_{m_1} \leq q \leq \bar{\beta}_{m_1}-1, 1 \leq r \leq \gamma_{n_1}-1} a_{l,m,n,p,q,r} \dot{x}_{p,q,r} \\ &+ \sum_{\alpha_{l_1} \leq p < \bar{\alpha}_{l_1}-1, \bar{\beta}_{m_1} \leq q \leq \infty, 1 \leq r \leq \gamma_{n_1}-1} a_{l,m,n,p,q,r} \dot{x}_{p,q,r} + \sum_{\bar{\alpha}_{l_1}-1 \leq p < \infty, \bar{\beta}_{m_1}-1 \leq q \leq \infty, \bar{\gamma}_{n_1}-1 \leq r \leq \infty} a_{l,m,n,p,q,r} \dot{x}_{p,q,r} \\ &+ H_{t,u,v} \sum_{\alpha_{l_t} \leq p < \bar{\alpha}_{l_t}, \beta_{m_u} \leq q \leq \bar{\beta}_{m_u}, \gamma_{n_v} \leq r \leq \bar{\gamma}_{n_v}} a_{l,m,n,p,q,r} x_{p,q,r} \\ &+ [H_{t,u,v+1} - H_{t,u,v}] \sum_{\bar{\alpha}_{l_t} \leq p < \bar{\alpha}_{l_t}, \beta_{m_u} \leq q \leq \bar{\beta}_{m_u}, \gamma_{n_v} \leq r \leq \gamma_{n_v}} a_{l,m,n,p,q,r} x_{p,q,r} \\ &+ [H_{t,u+1,v+1} - H_{t,u,v}] \sum_{\bar{\alpha}_{l_t} \leq p < \bar{\alpha}_{l_t}, \beta_{m_u} \leq q \leq \beta_{m_u}, \bar{\gamma}_{n_v} \leq r \leq \bar{\gamma}_{n_v}} a_{l,m,n,p,q,r} x_{p,q,r} \\ &+ [H_{t,u+1,v} - H_{t,u,v}] \sum_{\alpha_{l_t} \leq p < \bar{\alpha}_{l_t}, \bar{\beta}_{m_u} \leq q \leq \bar{\beta}_{m_u}, \gamma_{n_v} \leq r \leq \bar{\gamma}_{n_v}} a_{l,m,n,p,q,r} x_{p,q,r} \\ &+ [H_{t+1,u+1,v} - H_{t,u,v}] \sum_{\alpha_{l_t} \leq p < \bar{\alpha}_{l_t}, \beta_{m_u} \leq q \leq \beta_{m_u}, \bar{\gamma}_{n_v} \leq r \leq \bar{\gamma}_{n_v}} a_{l,m,n,p,q,r} x_{p,q,r} \\ &+ [H_{t+1,u,v} - H_{t,u,v}] \sum_{\alpha_{l_t} \leq p < \alpha_{l_t}, \bar{\beta}_{m_u} \leq q \leq \bar{\beta}_{m_u}, \gamma_{n_v} \leq r \leq \bar{\gamma}_{n_v}} a_{l,m,n,p,q,r} x_{p,q,r} \end{aligned}$$

$$\begin{aligned}
 &+ [H_{t+1,u,v+1} - H_{t,u,v}] \sum_{\alpha_l \leq p < \alpha_{l_t}, \beta_{m_u} \leq q \leq \bar{\beta}_{m_u}, \gamma_n \leq r \leq \gamma_{n_v}} a_{l,m,n,p,q,r} x_{p,q,r} \\
 &+ [H_{t+1,u+1,v+1} - H_{t,u,v}] \sum_{\bar{\alpha}_{l_t} \leq p < \bar{\alpha}_l, \bar{\beta}_{m_u} \leq q \leq \bar{\beta}_m, \bar{\gamma}_{n_v} \leq r \leq \bar{\gamma}_n} a_{l,m,n,p,q,r} x_{p,q,r}
 \end{aligned}$$

Absolute value properties gives us the following:

$$\begin{aligned}
 |\chi_{l,m,n}| &\leq \sum_{p=1, q=1, r=1}^{\alpha_l-1, \beta_m-1, \gamma_n-1} |a_{l,m,n,p,q,r}| |\acute{x}_{p,q,r}| + \sum_{1 \leq p < \alpha_l-1, \beta_m \leq q \leq \bar{\beta}_m-1, \bar{\gamma}_n \leq r \leq \infty} |a_{l,m,n,p,q,r}| |\acute{x}_{p,q,r}| \\
 &+ \sum_{1 \leq p < \alpha_l-1, \bar{\beta}_m \leq q \leq \infty, \gamma_n \leq r \leq \bar{\gamma}_n-1} |a_{l,m,n,p,q,r}| |\acute{x}_{p,q,r}| + \sum_{\alpha_l \leq p < \bar{\alpha}_l-1, 1 \leq q \leq \beta_m-1, \bar{\gamma}_n \leq r \leq \infty} |a_{l,m,n,p,q,r}| |\acute{x}_{p,q,r}| \\
 &+ \sum_{\bar{\alpha}_l \leq p < \infty, 1 \leq q \leq \beta_m-1, \gamma_n \leq r \leq \bar{\gamma}_n-1} |a_{l,m,n,p,q,r}| |\acute{x}_{p,q,r}| + \sum_{\bar{\alpha}_l \leq p < \infty, \beta_m \leq q \leq \bar{\beta}_m-1, 1 \leq r \leq \gamma_n-1} |a_{l,m,n,p,q,r}| |\acute{x}_{p,q,r}| \\
 &+ \sum_{\alpha_l \leq p < \bar{\alpha}_l-1, \bar{\beta}_m \leq q \leq \infty, 1 \leq r \leq \gamma_n-1} |a_{l,m,n,p,q,r}| |\acute{x}_{p,q,r}| + \sum_{\bar{\alpha}_l-1 \leq p < \infty, \bar{\beta}_m-1 \leq q \leq \infty, \bar{\gamma}_n-1 \leq r \leq \infty} |a_{l,m,n,p,q,r}| |\acute{x}_{p,q,r}| \\
 &+ |H_{t,u,v}| \sum_{\alpha_l \leq p < \bar{\alpha}_l, \beta_m \leq q \leq \bar{\beta}_m, \gamma_n \leq r \leq \bar{\gamma}_n} |a_{l,m,n,p,q,r}| |x_{p,q,r}| \\
 &+ |H_{t,u,v+1} - H_{t,u,v}| \sum_{\bar{\alpha}_{l_t} \leq p < \bar{\alpha}_l, \beta_{m_u} \leq q \leq \bar{\beta}_{m_u}, \gamma_n \leq r \leq \gamma_{n_v}} |a_{l,m,n,p,q,r}| |x_{p,q,r}| \\
 &+ |H_{t,u+1,v+1} - H_{t,u,v}| \sum_{\bar{\alpha}_{l_t} \leq p < \bar{\alpha}_l, \beta_m \leq q \leq \beta_{m_u}, \bar{\gamma}_{n_v} \leq r \leq \bar{\gamma}_n} |a_{l,m,n,p,q,r}| |x_{p,q,r}| \\
 &+ |H_{t,u+1,v} - H_{t,u,v}| \sum_{\alpha_{l_t} \leq p < \bar{\alpha}_{l_t}, \bar{\beta}_{m_u} \leq q \leq \bar{\beta}_m, \gamma_{n_v} \leq r \leq \bar{\gamma}_{n_v}} |a_{l,m,n,p,q,r}| |x_{p,q,r}| \\
 &+ |H_{t+1,u+1,v} - H_{t,u,v}| \sum_{\alpha_{l_t} \leq p < \bar{\alpha}_l, \beta_m \leq q \leq \beta_{m_u}, \bar{\gamma}_{n_v} \leq r \leq \bar{\gamma}_n} |a_{l,m,n,p,q,r}| |x_{p,q,r}| \\
 &+ |H_{t+1,u,v} - H_{t,u,v}| \sum_{\alpha_l \leq p < \alpha_{l_t}, \bar{\beta}_{m_u} \leq q \leq \bar{\beta}_m, \gamma_{n_v} \leq r \leq \bar{\gamma}_{n_v}} |a_{l,m,n,p,q,r}| |x_{p,q,r}| \\
 &+ |H_{t+1,u,v+1} - H_{t,u,v}| \sum_{\alpha_l \leq p < \alpha_{l_t}, \beta_{m_u} \leq q \leq \bar{\beta}_{m_u}, \gamma_n \leq r \leq \gamma_{n_v}} |a_{l,m,n,p,q,r}| |x_{p,q,r}| \\
 &+ |H_{t+1,u+1,v+1} - H_{t,u,v}| \sum_{\bar{\alpha}_{l_t} \leq p < \bar{\alpha}_l, \bar{\beta}_{m_u} \leq q \leq \bar{\beta}_m, \bar{\gamma}_{n_v} \leq r \leq \bar{\gamma}_n} |a_{l,m,n,p,q,r}| |x_{p,q,r}|
 \end{aligned}$$

The inequalities in condition (1) give the following:

$$|\chi_{l,m,n}| \leq 8U\delta_{l,m,n}$$

$$\begin{aligned}
& + |H_{t,u,v}| \Sigma_{\alpha_l \leq p < \bar{\alpha}_l, \beta_m \leq q \leq \bar{\beta}_m, \gamma_n \leq r \leq \bar{\gamma}_n} |a_{l,m,n,p,q,r}| |x_{p,q,r}| \\
& + |H_{t,u,v+1} - H_{t,u,v}| \Sigma_{\bar{\alpha}_l \leq p < \bar{\alpha}_l, \beta_{m_u} \leq q \leq \bar{\beta}_{m_u}, \gamma_n \leq r \leq \gamma_{n_v}} |a_{l,m,n,p,q,r}| |x_{p,q,r}| \\
& + |H_{t,u+1,v+1} - H_{t,u,v}| \Sigma_{\bar{\alpha}_l \leq p < \bar{\alpha}_l, \beta_m \leq q \leq \beta_{m_u}, \bar{\gamma}_{n_v} \leq r \leq \bar{\gamma}_n} |a_{l,m,n,p,q,r}| |x_{p,q,r}| \\
& + |H_{t,u+1,v} - H_{t,u,v}| \Sigma_{\alpha_l \leq p < \bar{\alpha}_l, \bar{\beta}_{m_u} \leq q \leq \bar{\beta}_m, \gamma_{n_v} \leq r \leq \bar{\gamma}_{n_v}} |a_{l,m,n,p,q,r}| |x_{p,q,r}| \\
& + |H_{t+1,u+1,v} - H_{t,u,v}| \Sigma_{\alpha_l \leq p < \bar{\alpha}_l, \beta_m \leq q \leq \beta_{m_u}, \bar{\gamma}_{n_v} \leq r \leq \bar{\gamma}_n} |a_{l,m,n,p,q,r}| |x_{p,q,r}| \\
& + |H_{t+1,u,v} - H_{t,u,v}| \Sigma_{\alpha_l \leq p < \alpha_{l_t}, \bar{\beta}_{m_u} \leq q \leq \bar{\beta}_m, \gamma_{n_v} \leq r \leq \bar{\gamma}_{n_v}} |a_{l,m,n,p,q,r}| |x_{p,q,r}| \\
& + |H_{t+1,u,v+1} - H_{t,u,v}| \Sigma_{\alpha_l \leq p < \alpha_{l_t}, \beta_{m_u} \leq q \leq \bar{\beta}_{m_u}, \gamma_n \leq r \leq \gamma_{n_v}} |a_{l,m,n,p,q,r}| |x_{p,q,r}| \\
& + |H_{t+1,u+1,v+1} - H_{t,u,v}| \Sigma_{\bar{\alpha}_l \leq p < \bar{\alpha}_l, \bar{\beta}_{m_u} \leq q \leq \bar{\beta}_m, \bar{\gamma}_{n_v} \leq r \leq \bar{\gamma}_n} |a_{l,m,n,p,q,r}| |x_{p,q,r}|
\end{aligned}$$

Then we have

$$|\acute{\chi}_{l,m,n}| \leq 8U\delta_{l,m,n} + \epsilon_{l,m,n} + 7V\lambda_{t,u,v}$$

$$\text{Similarly } |\acute{\psi}_{l,m,n}| \leq 8U\delta_{l,m,n} + \epsilon_{l,m,n} + 7V\lambda_{t,u,v}$$

Therefore $P - \lim_{l,m,n} (A\acute{\chi})_{l,m,n} = 0$ and $P - \lim_{l,m,n} (A\acute{\psi})_{l,m,n} = 0$

This completes the proof of the theorem. \square

Theorem 2.2. If $A = (a_{l,m,n,p,q,r})$ and $B = (b_{l,m,n,p,q,r})$ be two six dimensional RH -regular summability matrices sum a bounded triple sequence $(x_{p,q,r})$ to the different sums, then there exists a bounded triple sequence which is summed by A but not summed by B .

Proof. The proof of the theorem can be established following the technique applied in the above theorem. \square

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