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Skolem difference mean labeling of disconnected graphs

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Abstract

Let $G = (V, E)$ be a graph with p vertices and q edges. G is said to have skolem difference mean labeling if it is possible to label the vertices $x \in V$ with distinct elements $f(x)$ from $1, 2, 3, ..., p + q$ in such a way that for each edge $e = uv$, let $f^*(e) = \left\lceil \frac{|f(u)-f(v)|}{2} \right\rceil$ and the resulting labels of the edges are distinct and are from $1, 2, 3, ..., q$. A graph that admits a skolem difference mean labeling is called a skolem difference mean graph. In this paper, we prove that the graphs $C_m \cup C_n (n, m \geq 3$ and $m \leq n)$, $F_n \cup (n-2)K_2 (n > 2)$, $(P_n + \overline{K_2}) \cup (2n-3)K_2 (n \geq 2)$ and $W_n \cup (n-1)K_2 (n \geq 3)$ are skolem difference mean graphs.

Keywords: *mean labeling, skolem difference mean labeling, skolem difference mean graph.*

AMS Subject Classification: *05C78*

1. Introduction

Throughout this paper by a graph we mean a finite, simple and undirected one. The vertex set and the edge set of a graph G are denoted by $V(G)$ and $E(G)$ respectively. A graph labeling is an assignment of integers to the vertices or edges or both, subject to certain conditions. Terms and notations not defined here are used in the sense of Harary[1]. There are several types of labeling. An excellent survey of graph labeling is maintained by Gallian[2]. The notion of mean labeling was due to Somasundaram et al.[8]. A graph $G = (V, E)$ with p vertices and q edges is called a mean graph if there is an injective function f that maps $V(G)$ to $\{0, 1, 2, ..., q\}$ such that for each edge uv, labeled with $\frac{f(u)+f(v)}{2}$ if $f(u) + f(v)$ is even and $\frac{f(u)+f(v)+1}{2}$ if $f(u) + f(v)$ is odd. Then the resulting edge labels are distinct. The concept of skolem difference mean labeling was introduced by Murugan et al.[4] and they studied the skolem difference mean labeling of H-graphs. In[3], they studied skolem difference mean labeling of finite union of paths. Further, the skolem difference mean labeling of $K_n (n \geq 3)$, $K_{m,n}(m, n \geq 2)$, $G \cup \overline{K_n}$, $(G_1)_f * (G_2)_g$, $G \cup H$ were proved in[5]. Ramya et al. [6], proved that $\langle T\hat{o}K_{1,n}\rangle$, where T is a Tp-tree, caterpillar, $S_{m,n}$ and $C_n@K_{1,m}$ were skolem difference mean graphs. In [7], we proved that the graphs $C_n@P_m(n \geq 3, m \geq 1)$, $T\langle K_{1,n_1} : K_{1,n_2} : ... : K_{1,n_m}\rangle$, $T\langle K_{1,n_1} \circ K_{1,n_2} \circ \circ \circ K_{1,n_m}\rangle$, $st(n_1, n_2, ..., n_m)$ and $Bt(n, n, ...n)$ admit skolem difference mean labeling.

We use the following definitions in the subsequent section.

Definition 1.1. *A graph $G = (V, E)$ with p vertices and q edges is said to have skolem difference mean labeling if it is possible to label the vertices $x \in V$ with distinct elements $f(x)$ from $1, 2, 3, ..., p + q$ in such a way that for each edge $e = uv$, let $f^*(e) = \left\lceil \frac{|f(u)-f(v)|}{2} \right\rceil$ if $|f(u) - f(v)|$ is even and $\frac{|f(u)-f(v)|+1}{2}$ if $|f(u) - f(v)|$ is odd and the resulting labels of the edges are distinct and are from $1, 2, 3, ..., q$. A graph that admits a skolem difference mean labeling is called a skolem difference mean graph.*

Definition 1.2. *Let G_1 and G_2 be two graphs having vertex sets V_1 and V_2 and edge sets E_1 and E_2 respectively. Then their union $G_1 \cup G_2$ has $V = V_1 \cup V_2$ and $E = E_1 \cup E_2$.*

Definition 1.3. *A path is a walk if all the vertices and edges are distinct. A path on n vertices is denoted by P_n. The graph mP_n is the disjoint union of m copies of the path P_n.*

Definition 1.4. *The wheel graph W_n is obtained by joining a central vertex to all vertices of the cycle C_n by an edge.*

Definition 1.5. *The fan graph F_n is obtained by joining a vertex to all vertices of the path P_n by an edge.*

2. Main results

In this section we prove that $C_m \cup C_n(n, m \geq 3$ and $m \leq n)$, $F_n \cup (n-2)K_2(n > 2)$, $(P_n + \overline{K_2}) \cup (2n-3)K_2(n \geq 2)$ and $W_n \cup (n-1)K_2(n \geq 3)$ are skolem difference mean graphs.

Theorem 2.1. *The union of two cycles $C_m \cup C_n(n, m \geq 3$ and $m \leq n)$ is a skolem difference mean graph.*

Proof. **Case(i).** m is odd.
Let $m = 2l + 1$.

Subcase(i). n is odd.
Let $n = 2k + 1$
Let $u_1, u_2, .., u_l, v_l, v_{l-1}, ..., v_1, v_0$ be the vetices of C_{2l+1} and $u_1', u_2', ..., u_k', v_k', v_{k-1}', ..., v_1', v_0'$ be the vertices of C_{2k+1} respectively.

Then $E(C_m \cup C_n) = \{v_i v_{i+1} : 1 \leq i \leq l-1\} \cup \{u_i u_{i+1} : 1 \leq i \leq l-1\} \cup \{v_i' v_{i+1}' : 1 \leq i \leq k-1\} \cup \{u_i' u_{i+1}' : 1 \leq i \leq k-1\} \cup \{v_0 v_1, v_0 u_1, u_l v_l, v_0' v_1', v_0' u_1', u_k' v_k'\}$. Define $f : V(C_m \cup C_n) \rightarrow \{1, 2, 3, ..., p + q = 2(m+n)\}$ as follows:

$$f(v_0) = 1,$$
$$f(v_i) = \begin{cases} 2(m+n) - 2i + 1 & \text{if } i \text{ is odd and } 1 \leq i \leq l \\ 2i + 1 & \text{if } i \text{ is even and } 2 \leq i \leq l, \end{cases}$$
$$f(u_i) = \begin{cases} 2(m+n) - 2i + 2 & \text{if } i \text{ is odd and } 1 \leq i \leq l \\ 2i & \text{if } i \text{ is even and } 2 \leq i \leq l, \end{cases}$$

For $n = 3$, $f(v_0') = 2$, $f(v_1') = 7$, $f(u_1') = 10$.
When $n > 3$, $f(v_0') = 3$,

$$f(v'_i) = \begin{cases} 2n-2i+5 & \text{if } \; i \text{ is odd and } 1 \le i \le k-1 \\ 2i+3 & \text{if } \; i \text{ is even and } 2 \le i \le k-1, \end{cases}$$

$$f(u'_i) = \begin{cases} 2n-2i+6 & \text{if } \; i \text{ is odd and } 1 \le i \le k-1 \\ 2i+2 & \text{if } \; i \text{ is even and } 2 \le i \le k-1, \end{cases}$$

$$f(v'_k) = \begin{cases} n+7 & \text{if } \; k \text{ is odd} \\ n+1 & \text{if } \; k \text{ is even,} \end{cases}$$

$f(u'_k) = n+4.$

For each vertex label f, the induced edge label f^* is calculated as follows:

$f^*(v_iv_{i+1}) = m+n-2i-1$ for $1 \le i \le l-1$,
$f^*(u_iu_{i+1}) = m+n-2i$ for $1 \le i \le l-1$,
$f^*(v_0v_1) = n+m-1$, $f^*(u_1v_0) = n+m$, $f^*(u_lv_l) = 1$,
$f^*(v'_jv'_{j+1}) = n-2j$ for $1 \le j \le k-2$,
$f^*(u'_ju'_{j+1}) = n-2j+1$ for $1 \le j \le k-2$,
$f^*(v'_0v'_1) = n$, $f^*(v'_0u'_1) = n+1$, $f^*(v'_ku'_k) = 2$,
$f^*(u'_{k-1}u'_k) = 3$ and $f^*(v'_{k-1}v'_k) = 4$.

Therefore, $E(C_m \cup C_n) = \{1, 2, 3, ..., q\}$ and hence f is a skolem difference mean labeling.

Subcase(ii). n is even.

Let $n = 2k, k > 1$.

Let $u_1, u_2, .., u_l, v_l, v_{l-1}, ..., v_1, v_0$ be the vetices of C_{2l+1} and $u'_1, u'_2, ..., u'_{k-2}, u'_{k-1}, u'_0, v'_{k-1} ..., v'_1, v'_0$ be the vertices of C_{2k} respectively.

Then $E(C_m \cup C_n) = \{v_iv_{i+1} : 1 \le i \le l-1\} \cup \{u_iu_{i+1} : 1 \le i \le l-1\} \cup \{v'_jv'_{j+1} : 1 \le j \le k-2\} \cup \{u'_ju'_{j+1} : 1 \le j \le k-2\} \cup \{v_0v_1, v_0u_1, u_lv_l, v'_0v'_1, v'_0u'_1, u'_0u'_{k-1}, u'_0v'_{k-1}\}$.

Define $f : V(C_m \cup C_n) \to \{1, 2, 3, ..., p+q = 2(m+n)\}$ as follows:

$f(v_0) = 1,$

$$f(v_i) = \begin{cases} 2(m+n)-2i+1 & \text{if} \quad i \text{ is odd and } 1 \leq i \leq l \\ 2i+1 & \text{if} \quad i \text{ is even and } 2 \leq i \leq l, \end{cases}$$

$$f(u_i) = \begin{cases} 2(m+n)-2i+2 & \text{if} \quad i \text{ is odd and } 1 \leq i \leq l \\ 2i & \text{if} \quad i \text{ is even and } 2 \leq i \leq l, \end{cases}$$

$f(v_0^{'}) = 3,$

$$f(u_0^{'}) = \begin{cases} n+5 & \text{if} \quad k \text{ is odd} \\ n+3 & \text{if} \quad k \text{ is even}, \end{cases}$$

$$f(v_j^{'}) = \begin{cases} 2n-2j+5 & \text{if} \quad j \text{ is odd and } 1 \leq j \leq k-1 \\ 2j+3 & \text{if} \quad j \text{ is even and } 2 \leq j \leq k-1, \end{cases}$$

$$f(u_j^{'}) = \begin{cases} 2n-2j+6 & \text{if} \quad j \text{ is odd and } 1 \leq j \leq k-1 \\ 2j+2 & \text{if} \quad j \text{ is even and } 2 \leq j \leq k-1, \end{cases}$$

For each vertex label f, the induced edge label f^* is calculated as follows:

$f^*(v_iv_{i+1}) = m+n-2i-1$ for $1 \leq i \leq l-1$,
$f^*(u_iu_{i+1}) = m+n-2i$ for $1 \leq i \leq l-1$,
$f^*(v_0v_1) = n+m-1$, $f^*(u_1v_0) = n+m$, $f^*(u_lv_l) = 1$,
$f^*(v_j^{'}v_{j+1}^{'}) = n-2j$ for $1 \leq j \leq k-2$,
$f^*(u_j^{'}u_{j+1}^{'}) = n-2j+1$ for $1 \leq j \leq k-2$,
$f^*(u_0^{'}v_{k-1}^{'}) = 2$, $f^*(u_0^{'}u_{k-1}^{'}) = 3$, $f^*(v_0^{'}v_1^{'}) = n$ and $f^*(v_0^{'}u_1^{'}) = n+1$.

Therefore, we get the edge labels are from $\{1, 2, 3, ..., q\}$. Hence f is a skolem difference mean labeling.

Case(ii). m is even.

Let $m = 2l$.

Subcase(i). n is odd.

Let $n = 2k + 1$.

Let $u_1, u_2, .., u_{l-1}, u_0, v_{l-1}, ..., v_1, v_0$ be the vertices of C_{2l} and $u_1^{'}, u_2^{'}, ..., u_k^{'}, v_k^{'}, v_{k-1}^{'}, ..., v_1^{'}, v_0^{'}$ be the vertices of C_{2k+1} respectively.

Then $E(C_m \cup C_n) = \{v_i v_{i+1} : 1 \leq i \leq l-2\} \cup \{u_i u_{i+1} : 1 \leq i \leq l-2\} \cup \{v_j^{'} v_{j+1}^{'} : 1 \leq j \leq k-1\} \cup \{u_j^{'} u_{j+1}^{'} : 1 \leq j \leq k-1\} \cup \{v_0 v_1, v_0 u_1, u_{l-1} u_0, u_0^{'} v_{l-1}^{'}, v_0^{'} v_1^{'}, v_0^{'} u_1^{'}, u_k^{'} v_k^{'}\}$.

Define $f : V(C_m \cup C_n) \rightarrow \{1, 2, 3, ..., p + q = 2(m + n)\}$ as follows:

$f(v_0) = 1$,

$$f(v_i) = \begin{cases} 2(m+n) - 2i + 1 & \text{if} \quad i \text{ is odd and } 1 \leq i \leq l-1 \\ 2i + 1 & \text{if} \quad i \text{ is even and } 2 \leq i \leq l-1, \end{cases}$$

$$f(u_i) = \begin{cases} 2(m+n) - 2i + 2 & \text{if} \quad i \text{ is odd and } 1 \leq i \leq l-1 \\ 2i & \text{if} \quad i \text{ is even and } 2 \leq i \leq l-1, \end{cases}$$

$$f(u_0) = \begin{cases} m + 2n + 1 & \text{if} \quad l \text{ is odd} \\ m + 1 & \text{if} \quad l \text{ is even}, \end{cases}$$

$f(v_0^{'}) = 3$,

$$f(v_j^{'}) = \begin{cases} 2n - 2j + 3 & \text{if} \quad j \text{ is odd and } 1 \leq j \leq k \\ 2j + 3 & \text{if} \quad j \text{ is even and } 2 \leq j \leq k, \end{cases}$$

$$f(u_j^{'}) = \begin{cases} 2n - 2j + 4 & \text{if} \quad j \text{ is odd and } 1 \leq j \leq k \\ 2j + 2 & \text{if} \quad j \text{ is even and } 2 \leq j \leq k. \end{cases}$$

For each vertex label f, the induced edge label f^* is calculated as follows:

$f^*(v_i v_{i+1}) = m + n - 2i - 1$ for $1 \leq i \leq l-2$,
$f^*(u_i u_{i+1}) = m + n - 2i$ for $1 \leq i \leq l-2$,
$f^*(v_0 v_1) = n + m - 1$, $f^*(u_1 v_0) = n + m$,
$f^*(v_{l-1} u_0) = n + 1$, $f^*(u_0 u_{l-1}) = n + 2$,
$f^*(v_j^{'} v_{j+1}^{'}) = n - 2j - 1$ for $1 \leq j \leq k-1$,
$f^*(u_j^{'} u_{j+1}^{'}) = n - 2j$ for $1 \leq j \leq k-1$,

$f^*(v_0^{'}v_1^{'}) = n-1,\ f^*(v_0^{'}u_1^{'}) = n,\ f^*(v_k^{'}u_k^{'}) = 1.$

Therefore, f is a skolem difference mean labeling.

Subcase(ii). n is even.

Let $n = 2k, k > 1$.

Let $u_1, u_2, .., u_{l-1}, u_o, v_{l-1}, ..., v_1, v_0$ be the vertices of C_{2l} and $u_1^{'}, u_2^{'}, ..., u_{k-2}^{'}, u_{k-1}^{'}, u_0^{'}, v_{k-1}^{'} ..., v_1^{'}, v_0^{'}$ be the vertices of C_{2k} respectively.

Then $E(C_m \cup C_n) = \{v_iv_{i+1} : 1 \leq i \leq l-2\} \cup \{u_iu_{i+1} : 1 \leq i \leq l-2\} \cup \{v_j^{'}v_{j+1}^{'} : 1 \leq j \leq k-2\} \cup \{u_j^{'}u_{j+1}^{'} : 1 \leq j \leq k-2\} \cup \{v_0v_1, v_0u_1, u_0v_{l-1}, u_0u_{l-1}, v_0^{'}v_1^{'}, v_0^{'}u_1^{'}, u_0^{'}u_{k-1}^{'}, u_0^{'}v_{k-1}^{'}\}$.

Define $f : V(C_m \cup C_n) \to \{1, 2, 3, ..., p+q = 2(m+n)\}$ as follows:

$f(v_0) = 1,$

$$f(v_i) = \begin{cases} 2(m+n) - 2i + 1 & \text{if} \quad i \text{ is odd and } 1 \leq i \leq l-1 \\ 2i + 1 & \text{if} \quad i \text{ is even and } 2 \leq i \leq l-1, \end{cases}$$

$$f(u_i) = \begin{cases} 2(m+n) - 2i + 2 & \text{if} \quad \text{i is odd and } 1 \leq i \leq l-1 \\ 2i & \text{if} \quad \text{i is even and } 1 \leq i \leq l-1, \end{cases}$$

$$f(u_0) = \begin{cases} m + 2n + 1 & \text{if} \quad l \text{ is odd} \\ m + 1 & \text{if} \quad l \text{ is even}, \end{cases}$$

$f(v_0^{'}) = 3,$

$f(u_0^{'}) = n + 3$

$$f(v_j^{'}) = \begin{cases} 2n - 2j + 3 & \text{if} \quad j \text{ is odd and } 1 \leq j \leq k-1 \\ 2j + 3 & \text{if} \quad j \text{ is even and } 1 \leq j \leq k-1, \end{cases}$$

$$f(v_{k-1}^{'}) = \begin{cases} n + 2 & \text{if} \quad \text{k is odd} \\ n + 4 & \text{if} \quad \text{k is even} \end{cases}$$

$$f(u_j^{'}) = \begin{cases} 2n-2j+4 & \text{if} \quad j \text{ is odd and } 1 \le j \le k-1 \\ 2j+2 & \text{if} \quad j \text{ is even and } 2 \le j \le k-1. \end{cases}$$

For each vertex label f, the induced edge label f^* is calculated as follows:

$f^*(v_iv_{i+1}) = m+n-2i-1$ for $1 \le i \le l-2$,
$f^*(u_iu_{i+1}) = m+n-2i$ for $1 \le i \le l-2$,
$f^*(v_0v_1) = n+m-1$, $f^*(u_1v_0) = n+m$, $f^*(u_0v_{l-1}) = n+1$, $f^*(u_0u_{l-1}) = n+2$,
$f^*(v_j^{'}v_{j+1}^{'}) = n-2j-1$ for $1 \le j \le k-2$,
$f^*(u_j^{'}u_{j+1}^{'}) = n-2j$ for $1 \le j \le k-2$,
$f^*(v_0^{'}v_1^{'}) = n-1$, $f^*(v_0^{'}u_1^{'}) = n$, $f^*(v_{k-1}^{'}u_0^{'}) = 1$, $f^*(u_0^{'}u_{k-1}^{'}) = 2$.

Therefore, f is a skolem difference mean labeling and hence $C_m \cup C_n$ is a skolem difference mean graph. The skolem difference mean labeling of $C_7 \cup C_8$ and $C_9 \cup C_3$ are shown in Figure 1 and 2.

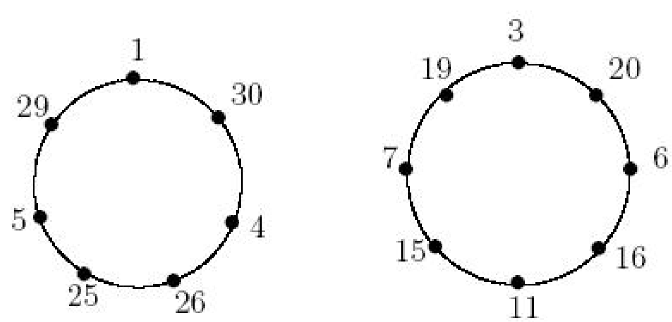

Figure 1: $C_7 \cup C_8$

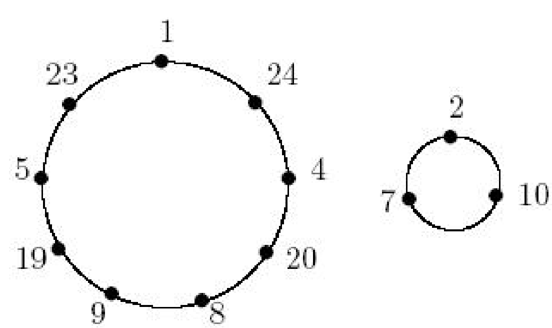

Figure 2: $C_9 \cup C_3$

Theorem 2.2. *The graph $F_n \cup (n-2)K_2, (n > 2)$ is a skolem difference mean graph.*

Proof. Let v_0, v_i $(1 \le i \le n)$ be the vertices of F_n and x_j, y_j $(1 \le j \le n-2)$ be the vertices of $(n-2)K_2$ respectively.

Then $E(F_n \cup (n-2)K_2) = \{v_i v_{i+1} : 1 \le i \le n-1\} \cup \{v_0 v_i : 1 \le i \le n\} \cup \{x_j y_j : 1 \le j \le (n-2)\}$.

Define $f : V(F_n \cup (n-2)K_2) \to \{1, 2, 3, ..., p+q = 6(n-1)\}$ as follows:

$f(v_0) = 5n - 4$,
$f(v_i) = n - 2 + i$ if i is odd and $1 \le i \le n$,
$f(v_i) = 3n - 2 - i$ if i is even and $2 \le i \le n$,
$f(x_j) = j$ for $1 \le j \le n-2$,
$f(y_j) = 6n - 5 - j$ for $1 \le j \le n-2$.

Let $e_i = v_0 v_i$, $e_i' = v_i v_{i+1} (1 \le i \le n-1)$ and $e_j = x_j y_j (1 \le j \le n-2)$.

For each vertex label f, the induced edge label f^* is calculated as follows:

$f^*(e_j) = 3n - 2 - j$ for $1 \le j \le n-2$,
$f^*(e_i') = n - i$ for $1 \le i \le n-1$,
$f^*(e_i) = 2n - (\frac{i+1}{2})$ if i is odd and $1 \le i \le n$,
$f^*(e_i) = n + (\frac{i-2}{2})$ if i is even and $2 \le i \le n$.

Therefore, f is a skolem difference mean labeling of $F_n \cup (n-2)K_2$ and hence $F_n \cup (n-2)K_2 (n > 2)$ is a skolem difference mean graph.

A skolem difference mean labeling of $F_7 \cup 5K_2$ is shown in Figure 3.

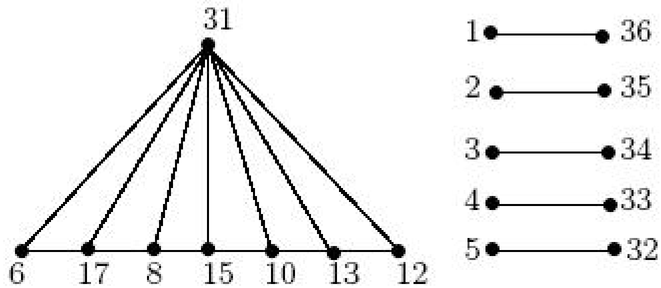

Figure 3: $F_7 \cup 5K_2$

Theorem 2.3. *The graph* $(P_n + \overline{K_2}) \cup (2n-3)K_2, (n \geq 2)$ *is a skolem difference mean graph.*

Proof. Let u_0, v_0, v_i $(1 \leq i \leq n)$ be the vertices of $(P_n + \overline{K_2})$ and x_t, y_t $(1 \leq t \leq 2n-3)$ be the vertices of $(2n-3)K_2$ respectively.

Then $E((P_n + \overline{K_2}) \cup (2n-3)K_2) = \{v_i v_{i+1} : 1 \leq i \leq n-1\} \cup \{u_0 v_i, v_0 v_i : 1 \leq i \leq n\} \cup \{x_t y_t : 1 \leq t \leq (2n-3)\}$.

Define $f : V((P_n + \overline{K_2}) \cup (2n-3)K_2) \rightarrow \{1, 2, 3, ..., p+q = 10n-8\}$ as follows:

$f(v_0) = 8n-5$, $f(u_0) = 6n-5$,
$f(v_i) = 2n-3+i$ if i is odd and $1 \leq i \leq n$,
$f(v_i) = 4n-2-i$ if i is even and $2 \leq i \leq n$,
$f(x_t) = t$ for $1 \leq t \leq 2n-3$,
$f(y_t) = 10n-7-t$ for $1 \leq t \leq 2n-3$.

Let $e_i = v_0 v_i$, $e_i' = u_0 v_i (1 \leq i \leq n)$, $e_j = v_j v_{j+1} (1 \leq j \leq n-1)$ and $e_t = x_t y_t (1 \leq t \leq 2n-3)$.

For each vertex label f, the induced edge label f^* is calculated as follows:

$f^*(e_t) = 5n-3-t$ for $1 \leq t \leq 2n-3$,
$f^*(e_j) = n-j$ for $1 \leq j \leq n-1$,

$f^*(e_i) = 3n - (\frac{i+1}{2})$ if i is odd and $1 \leq i \leq n$,
$f^*(e_i) = 2n + (\frac{i-2}{2})$ if i is even and $1 \leq i \leq n$,
$f^*(e_i^{'}) = 2n - (\frac{i+1}{2})$ if i is odd and $1 \leq i \leq n$,
$f^*(e_i^{'}) = n + (\frac{i-2}{2})$ if i is even and $1 \leq i \leq n$.

Therefore, f is a skolem difference mean labeling of $(P_n + \overline{K_2}) \cup (2n - 3)K_2$ and hence $(P_n + \overline{K_2}) \cup (2n-3)K_2$ is a skolem difference mean graph.

A skolem difference mean labeling of $(P_7 + \overline{K_2}) \cup 11K_2$ is shown in Figure 4.

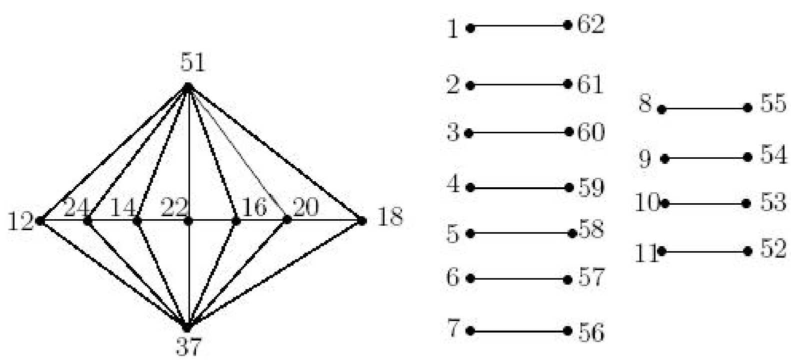

Figure 4: $(P_7 + \overline{K_2}) \cup 11K_2$

Theorem 2.4. *The graph $W_n \cup (n-1)K_2, (n \geq 3)$ is a skolem difference mean graph.*

Proof. Let v_0, v_i $(1 \leq i \leq n)$ be the vertices of W_n and x_t, y_t $(1 \leq t \leq n-1)$ be the vertices of $(n-1)K_2$ respectively.

Then $E(W_n \cup (n-1)K_2) = \{v_i v_{i+1} : 1 \leq i \leq n-1\} \cup \{v_0 v_i : 1 \leq i \leq n\} \cup \{x_t y_t : 1 \leq t \leq (n-1)\} \cup \{v_n v_1\}$.

Define $f : V(W_n \cup (n-1)K_2) \rightarrow \{1, 2, 3, ..., p + q = 6n - 2\}$ as follows:

Case(i). n is odd.

Let $n = 2k + 1$.
$f(x_t) = t$ for $1 \leq t \leq n-2$,
$f(x_{n-1}) = n$,
$f(y_t) = 6n - t - 1$ for $1 \leq t \leq n-2$,
$f(y_{n-1}) = 5n - 1,\ f(v_0) = 5n,\ f(v_1) = n - 1$,
$f(v_i) = 3n - 2i + 2$ if i is even and $2 \leq i \leq n-1$,
$f(v_i) = n + 2i - 4$ if i is odd and $3 \leq i \leq k+1$,
$f(v_{2k+2-i}) = 3n - 2i - 1$ if i is odd and $1 \leq i \leq k$.

For each vertex label f, the induced edge label f^* is calculated as follows:

$f^*(x_t y_t) = 3n - t$ for $1 \leq t \leq n-2$,
$f^*(x_{n-1} y_{n-1}) = 2n$,
$f^*(v_i v_{i+1}) = n - 2i + 2$ for $1 \leq i \leq k+1$,
$f^*(v_i v_{i+1}) = 2i - n - 1$ for $k+2 \leq i \leq 2k$,
$f^*(v_{2k+1} v_1) = n - 1$,
$f^*(v_0 v_i) = 2n + 2 - i$ if i is odd and $1 \leq i \leq n$,
$f^*(v_0 v_i) = n + i - 1$ if i is even and $2 \leq i \leq n$.

Therefore, $E(W_n \cup (n-1)K_2) = \{1, 2, 3, ..., q\}$ and hence f is a skolem difference mean labeling.

Case(ii). n is even.

Let $n = 2k, k > 1$.
$f(x_t) = t$ for $1 \leq t \leq n-2$,
$f(x_{n-1}) = n$,
$f(y_t) = 6n - t - 1$ for $1 \leq t \leq n-2$,
$f(y_{n-1}) = 5n - 1,\ f(v_0) = 5n,\ f(v_1) = n - 1$,
$f(v_i) = 3n - 2i + 2$ if i is even and $2 \leq i \leq k$,
$f(v_i) = n + 2i - 4$ if i is odd and $3 \leq i \leq k$,

$$\text{f}(\text{v}_{k+1}) = \begin{cases} 2n-1 & \text{if} \quad k \text{ is even} \\ 2n & \text{if} \quad k \text{ is odd,} \end{cases}$$

$f(v_{2k+1-i}) = 3n - 2i - 1$ if i is odd and $1 \leq i \leq k-1$,
$f(v_{2k+1-i}) = n + 2i$ if i is even and $2 \leq i \leq k-1$.

For each vertex label f, the induced edge label f^* is calculated as follows:

$f^*(x_t y_t) = 3n - t$ for $1 \leq t \leq n-2$,
$f^*(x_{n-1} y_{n-1}) = 2n$,
$f^*(v_i v_{i+1}) = n - 2i + 2$ for $1 \leq i \leq k$,
$f^*(v_i v_{i+1}) = 2i - n - 1$ for $k+1 \leq i \leq 2k-1$,
$f^*(v_{2k} v_1) = n - 1$,

$$f^*(v_0 v_i) = \begin{cases} 2n+2-i & \text{if } i \text{ is odd and } 1 \leq i \leq k+1 \\ n-1+i & \text{if } i \text{ is even and } 2 \leq i \leq k+1, \end{cases}$$

$$f^*(v_0 v_{2k+1-i}) = \begin{cases} n+1+i & \text{if } i \text{ is odd and } 1 \leq i \leq k-1 \\ 2n-i & \text{if } i \text{ is even and } 2 \leq i \leq k-1. \end{cases}$$

Therefore, f is a skolem difference mean labeling and hence $W_n \cup (n-1)K_2$ is a skolem difference mean graph.

A skolem difference mean labeling of $W_8 \cup 7K_2$ is shown in Figure 5.

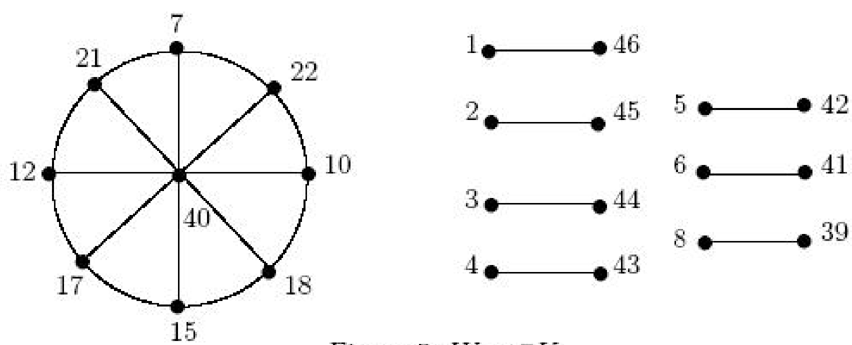

Figure 5: $W_8 \cup 7K_2$

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