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On some maps concerning $g\beta\theta$ -open sets

Miguel Caldas

Universidade Federal Fluminense, Brasil

and

Saeid Jafari

College of Vestsjaelland South, Denmark

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Abstract

In this paper, we consider a new notion of $\beta\theta$ -open maps via the concept of $g\beta\theta$ -closed sets which we call approximately $\beta\theta$ -open maps. We study some of its fundamental properties. It turns out that we can use this notion to obtain a new characterization of $\beta\theta$ - $T_{\frac{1}{2}}$ spaces.

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1. Introduction and Preliminaries

In 2003, T. Noiri [10] introduced a new class of sets called $\beta\theta$ -open in topological space. In M. Caldas [4, 5], a $\beta\theta$ -open function is defined as follows: A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is $\beta\theta$ -open if $f(A)$ is $\beta\theta$ -open in Y for

open set A of X , and weakly $\beta\theta$ -open functions which is independent with weakly open functions given by D.A. Rose [11]. In this direction, we introduce the notion of ap- $\beta\theta$ -open maps by using $g\beta\theta$ -closed sets and study some of their basic properties. Finally we characterize the class of $\beta\theta$ - $T_{\frac{1}{2}}$ spaces in terms of ap- $\beta\theta$ -open maps.

Throughout the present paper, spaces (X, τ) and (Y, σ) (or simply, X and Y) denote topological spaces on which no separation axioms are assumed unless explicitly stated. Abd. El Monsef et al. [1] and Andrijević [2] introduced the notion of β -open set, which Andrijević called semipreopen, completely independent of each other. In this paper, we adopt the word β -open for the sake of clarity. A subset A of a topological space (X, τ) is called β -open if $A \subseteq \text{Cl}(\text{Int}(\text{Cl}(A)))$, where $\text{Cl}(A)$ and $\text{Int}(A)$ denote the closure and the interior of A , respectively. The complement of a β -open set is called β -closed. The family of all β -open sets of a space X is denoted by $\beta O(X, \tau)$ or $\beta O(X)$. We set $\beta O(X, x) = \{U : x \in U \in \beta O(X)\}$. The intersection of all β -closed sets containing A is called the β -closure of A and is denoted by $\beta \text{Cl}(A)$. The β -interior of a subset $A \subset X$, denoted by $\beta \text{Int}(A)$, is the union of all β -open sets contained in A .

Now we recall the well-known notions given by T.Noiri [10] which will be used in the sequel.

Definition 1. [10]. Let A a subset of X . The β - θ -closure of A , denoted by $\beta \text{Cl}_\theta(A)$, is the set of all $x \in X$ such that $\beta \text{Cl}(O) \cap A \neq \emptyset$ for every $O \in \beta O(X, x)$. A subset A is called β - θ -closed if $A = \beta \text{Cl}_\theta(A)$. The set $\{x \in X \mid \beta \text{Cl}_\theta(O) \subset A \text{ for some } O \in \beta O(X, x)\}$ is called the β - θ -interior of A and is denoted by $\beta \text{Int}_\theta(A)$. A subset A is called β - θ -open if $A = \beta \text{Int}_\theta(A)$.

Theorem 1.1. [10]. For any subset A of X :

- (1) $\beta \text{Cl}_\theta(\beta \text{Cl}_\theta(A)) = \beta \text{Cl}_\theta(A)$.
- (2) $\beta \text{Cl}_\theta(A)$ is $\beta\theta$ -closed.
- (3) Intersection of arbitrary collection of $\beta\theta$ -closed set in X is $\beta\theta$ -closed.

- (4) $\beta Cl_{\theta}(A)$ is the intersection of all $\beta\theta$ -closed sets each containing A .
 (5) If $A \in \beta O(X)$ then, $\beta Cl(A) = \beta Cl_{\theta}(A)$.

A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is called β -irresolute [10] if $f^{-1}(O)$ is β -open in (X, τ) for every $O \in \beta O(Y, \sigma)$.

A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is called pre- $\beta\theta$ -closed (resp. pre- $\beta\theta$ -open) [6, 7] if for every $\beta\theta$ -closed (resp. $\beta\theta$ -open) set B of (X, τ) , $f(B)$ is $\beta\theta$ -closed (resp. $\beta\theta$ -open) in (Y, σ) .

For other advances on topological spaces obtained by our research group we recommend [3, 4, 8, 10].

2. Ap- $\beta\theta$ -open maps.

We introduce the following notion:

Definition 2. A subset A of a topological space (X, τ) is called generalized $\beta\theta$ -closed (briefly $g\beta\theta$ -closed) if $\beta Cl_{\theta}(A) \subset O$ whenever $A \subset O$ and $O \in \beta\theta O(X, \tau)$.

Lemma 2.1. (i) Every $\beta\theta$ -closed set is $g\beta\theta$ -closed.
 (ii) [10] Every $\beta\theta$ -closed set is β -closed.
 But not conversely.

Proof. (i) Let $A \subset X$ be $\beta\theta$ -closed. Then $A = \beta Cl_{\theta}(A)$. Let $A \subset O$ and $O \in \beta\theta O(X, \tau)$. It follows that $\beta Cl_{\theta}(A) \subset O$. This means that A is $g\beta\theta$ -closed.

(ii) See Remark 3.2 in [10].

Example 2.2. Let $X = \{a, b, c\}$ and $\tau = \{\emptyset, \{a\}, \{a, b\}, X\}$. If $A = \{a, b\}$. Then $\beta Cl(A) = X$ and so A is not β -closed. Hence A is not $\beta\theta$ -closed. Since X is the only $\beta\theta$ -open containing A . A is $g\beta\theta$ -closed.

Example 2.3. Let $X = \{a, b, c\}$ and $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, X\}$. If $A = \{c\}$, then A is a β -closed set which is not $\beta\theta$ -closed.

Recall that, the complement of a generalized $\beta\theta$ -closed set is called generalized $\beta\theta$ -open (briefly $g\beta\theta$ -open).

Theorem 2.4. A set $A \subset (X, \tau)$ is $g\beta\theta$ -open if and only if $F \subset \beta Int_{\theta}(A)$ whenever F is $\beta\theta$ -closed in X and $F \subset A$.

Proof. Necessity. Let A be $g\beta\theta$ -open and $F \subset A$, where F is $\beta\theta$ -closed. It is obvious that A^c is contained in F^c . This implies that $\beta Cl_\theta(A^c) \subset F^c$. Hence $\beta Cl_\theta(A^c) = (\beta Int_\theta(A))^c \subset F^c$, i.e. $F \subset \beta Int_\theta(A)$.

Sufficiency. If F is a $\beta\theta$ -closed set with $F \subset \beta Int_\theta(A)$ whenever $F \subset A$, then it follows that $A^c \subset F^c$ and $(\beta Int_\theta(A))^c \subset F^c$ i.e., $\beta Cl_\theta(A^c) \subset F^c$. Therefore A^c is $g\beta\theta$ -closed and therefore A is $g\beta\theta$ -open. Hence the proof. \square

Definition 3. A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be approximately $\beta\theta$ -open (briefly ap- $\beta\theta$ -open) if $\beta Cl_\theta(B) \subseteq f(A)$ whenever B is a $g\beta\theta$ -closed subset of (Y, σ) , A is a $\beta\theta$ -open subset of (X, τ) and $B \subseteq f(A)$.

Theorem 2.5. $f : (X, \tau) \rightarrow (Y, \sigma)$ is ap- $\beta\theta$ -open if $f(O) \in \beta\theta C(Y, \sigma)$ for every $\beta\theta$ -open subset O of (X, τ) .

Proof. Let $B \subseteq f(A)$, where A is a $\beta\theta$ -open subset of (X, τ) and B is a $g\beta\theta$ -closed subset of (Y, σ) . Therefore $\beta Cl_\theta(B) \subseteq \beta Cl_\theta(f(A)) = f(A)$. Thus f is ap- $\beta\theta$ -open. \square

Clearly pre- $\beta\theta$ -open maps are ap- $\beta\theta$ -open, but not conversely.

Example 2.6. Let $X = \{a, b\}$ be the Sierpinski space with the topology, $\tau = \{\emptyset, \{a\}, X\}$. Let $f : X \rightarrow X$ be defined by $f(a) = b$ and $f(b) = a$. Since the image of every $\beta\theta$ -open set is $\beta\theta$ -closed, then f is ap- $\beta\theta$ -open. However $\{a\}$ is $\beta\theta$ -open in (X, τ) but $f(\{a\})$ is not $\beta\theta$ -open in (Y, σ) . Therefore f is not pre- $\beta\theta$ -open.

Remark 2.7. It is clear that the converse of Theorem 2.5 does not hold: Let (X, τ) be the topological space defined in Example 2.6. Then the identity map on (X, τ) is ap- $\beta\theta$ -open, but $f(a) \notin \beta\theta C(X, \tau)$ for $\{a\}$ a $\beta\theta$ -open set of (X, τ) .

In the following theorem, we show that under certain conditions the converse of Theorem 2.5 is true.

Theorem 2.8. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a map. If the $\beta\theta$ -open and $\beta\theta$ -closed sets of (Y, σ) coincide, then f is ap- $\beta\theta$ -open if and only if $f(A) \in \beta\theta C(Y, \sigma)$ for every $\beta\theta$ -open subset A of (X, τ) .

Proof. Suppose that f is ap- $\beta\theta$ -open. Let A be an arbitrary subset of (Y, σ) such that $A \subseteq Q$ where $Q \in \beta\theta O(Y, \sigma)$. Then by hypothesis

$\beta Cl_\theta(A) \subseteq \beta Cl_\theta(Q) = Q$. Therefore all subset of (Y, σ) are $g\beta\theta$ -closed and hence all are $g\beta\theta$ -open. So for any $O \in \beta\theta O(X, \tau), f(O)$ is $g\beta\theta$ -closed in (Y, σ) . Since f is $ap\text{-}\beta\theta$ -open $\beta Cl_\theta(f(O)) \subseteq f(O)$. Therefore $\beta Cl_\theta(f(O)) = f(O)$, i.e., $f(O)$ is $\beta\theta$ -closed in (Y, σ) . The converse is obvious by Theorem 2.5. \square

As an immediate consequence of Theorem 2.8, we have the following:

Corollary 2.9. *Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a map. If the $\beta\theta$ -closed and $\beta\theta$ -open sets of (Y, σ) coincide, then f is $ap\text{-}\beta\theta$ -open if and only if, f is $pre\text{-}\beta\theta$ -open.*

Definition 4. *A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is called *contra $pre\text{-}\beta\theta$ -open* if $f(O)$ is $\beta\theta$ -closed in (Y, σ) for each set $O \in \beta\theta O(X, \tau)$.*

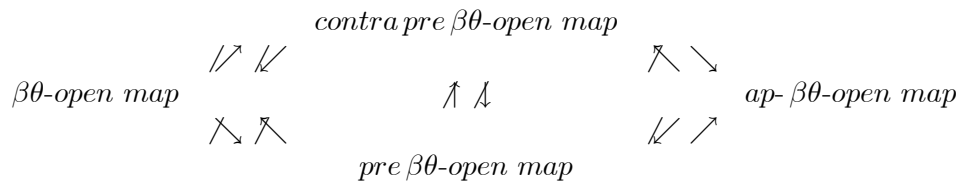
Remark 2.10. *In fact contra $pre\text{-}\beta\theta$ -open maps and $pre\text{-}\beta\theta$ -open are independent notions.*

Example 2.6 above shows that contra- $pre\text{-}\beta\theta$ -openness does not imply $pre\text{-}\beta\theta$ -openness while the converse is shown in the following example.

Example 2.11. *The identity map on the same topological space (X, τ) where $\tau = \{\emptyset, \{a\}, X\}$ is an example of a $pre\text{-}\beta\theta$ -open map which is not contra $pre\text{-}\beta\theta$ -open.*

Remark 2.12. *By Theorem 2.5 and Remark 2.7, we have that every contra $pre\text{-}\beta\theta$ -open map is $ap\text{-}\beta\theta$ -open, the converse does not hold.*

The following diagram holds :



The next theorem establishes conditions under which the inverse map of every $g\beta\theta$ -open set from codomain is a $g\beta\theta$ -open set.

Theorem 2.13. *If a map $f : (X, \tau) \rightarrow (Y, \sigma)$ is surjective β -irresolute and ap- $\beta\theta$ -open, then $f^{-1}(A)$ is $g\beta\theta$ -open whenever A is $g\beta\theta$ -open subset of (Y, σ) .*

Proof. Let A be a $g\beta\theta$ -open subset of (Y, σ) . Suppose that $F \subseteq f^{-1}(A)$, where $F \in \beta\theta C(X, \tau)$. Taking complements we obtain $f^{-1}(A^c) \subseteq F^c$ or $A^c \subseteq f(F^c)$. Since f is an ap- $\beta\theta$ -open and $(\beta Int_{\theta}(A))^c = \beta Cl_{\theta}(A^c) \subseteq f(F^c)$. It follows that $(f^{-1}(\beta Int_{\theta}(A)))^c \subseteq F^c$ and hence $F \subseteq f^{-1}(\beta Int_{\theta}(A))$. Since f is β -irresolute $f^{-1}(\beta Int_{\theta}(A))$ is $\beta\theta$ -open. Thus, we have $F \subseteq f^{-1}(\beta Int_{\theta}(A)) = \beta Int_{\theta}(f^{-1}(\beta Int_{\theta}(A))) \subseteq \beta Int_{\theta}(f^{-1}(A))$. This implies that $f^{-1}(A)$ is $g\beta\theta$ -open in (X, τ) . \square

A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is called $g\beta\theta$ -irresolute if $f^{-1}(O)$ is $g\beta\theta$ -closed in (X, τ) for every $g\beta\theta$ -closed in (Y, σ) .

Theorem 2.14. *Let $f : (X, \tau) \rightarrow (Y, \sigma)$, $g : (Y, \sigma) \rightarrow (Z, \gamma)$ be two maps such that $g \circ f : (X, \tau) \rightarrow (Z, \gamma)$. Then*

- (i) $g \circ f$ is ap- $\beta\theta$ -open, if f is pre- $\beta\theta$ -open and g is ap- $\beta\theta$ -open.
- (ii) $g \circ f$ is ap- $\beta\theta$ -open, if f is ap- $\beta\theta$ -open and g is bijective pre- $\beta\theta$ -closed and $g\beta\theta$ -irresolute.

Proof. (i) Suppose that A is an arbitrary $\beta\theta$ -open subset in (X, τ) and B a $g\beta\theta$ -closed subset of (Z, γ) for which $B \subseteq g \circ f(A)$. Then $f(A)$ is $\beta\theta$ -open in (Y, σ) since f is pre- $\beta\theta$ -open. Since g is ap- $\beta\theta$ -open, $\beta Cl_{\theta}(B) \subseteq g(f(A))$. This implies that $g \circ f$ is ap- $\beta\theta$ -open.

(ii) Let A be an arbitrary $\beta\theta$ -open subset of (X, τ) and B a $g\beta\theta$ -closed subset of (Z, γ) for which $B \subseteq g \circ f(A)$. Hence $g^{-1}(B) \subseteq f(A)$. Then $\beta Cl_{\theta}(g^{-1}(B)) \subseteq f(A)$ because $g^{-1}(B)$ is $g\beta\theta$ -closed and f is ap- $\beta\theta$ -open. Hence we have ,

$$\beta Cl_{\theta}(B) \subseteq \beta Cl_{\theta}(gg^{-1}(B)) \subseteq g(\beta Cl_{\theta}(g^{-1}(B))) \subseteq g(f(A)) = (g \circ f)(A).$$

This implies that $g \circ f$ is ap- $\beta\theta$ -open. \square

Now we state the following two theorems whose proof are straightforward and hence omitted.

Theorem 2.15. *Let $f : (X, \tau) \rightarrow (Y, \sigma)$ and $g : (Y, \sigma) \rightarrow (Z, \gamma)$ be two maps such that*

$g \circ f : (X, \tau) \rightarrow (Z, \gamma)$. Then,

- (i) $g \circ f$ is contra pre $\beta\theta$ -open, if f is pre $\beta\theta$ -open and g is contra pre $\beta\theta$ -open.
- (ii) $g \circ f$ is contra pre $\beta\theta$ -open, if f is contra pre $\beta\theta$ -open and g is pre $\beta\theta$ -closed.

Theorem 2.16. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ and $g : (Y, \sigma) \rightarrow (Z, \gamma)$ be two maps such that $g \circ f : (X, \tau) \rightarrow (Z, \gamma)$ is contra pre $\beta\theta$ -open.

(i) If f is a β -irresolute surjection, then g is contra pre $\beta\theta$ -open.

(ii) If g is a β -irresolute injection, then f is contra pre $\beta\theta$ -open .

Definition 5. Let (X, τ) and (Y, σ) be topological spaces. A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to have an $\beta\theta$ -closed graph if its $Gr(f) = \{(x, y) : y = f(x), x \in X\}$ is $\beta\theta$ -closed in the product space $(X \times Y, \tau_p)$, where τ_p denotes the product topology.

We say that the product space $X = X_1 \times \dots \times X_n$ has Property $P_{\beta\theta}$ if A_i is a $\beta\theta$ -open set in a topological spaces X_i , for $i = 1, 2, \dots, n$, then $A_1 \times \dots \times A_n$ is also $\beta\theta$ -open in the product space $X = X_1 \times \dots \times X_n$.

It is well-known that the graph $Gr(f)$ of f is a closed set of $X \times Y$, whenever f is continuous and Y is Hausdorff. The following theorem is a modification of this fact, i.e., we give a condition under which a contra pre $\beta\theta$ -open map has $\beta\theta$ -closed graph.

Theorem 2.17. If $f : (X, \tau) \rightarrow (Y, \sigma)$ is a contra pre $\beta\theta$ -open map with $\beta\theta$ -closed fibers, then the graph $Gr(f)$ of f is $\beta\theta$ -closed in the product space $X \times Y$. which have the property $P_{\beta\theta}$.

Proof. Let $(x, y) \in X \times Y \setminus Gr(f)$. Then $x \in (f^{-1}\{y\})^c$. Since fibers are $\beta\theta$ -closed, there is an $\beta\theta$ -open set O for which $x \in O \subseteq (f^{-1}\{y\})^c$. Set $A = (f(O))^c$. Then A is an $\beta\theta$ -open set in Y containing y , since f is contra pre $\beta\theta$ -open. Therefore, we obtain that $(x, y) \in O \times A \subseteq X \times Y \setminus Gr(f)$, where $O \times A \in \beta\theta O(X \times Y, \tau_p)$. This implies that $X \times Y \setminus Gr(f)$ is $\beta\theta$ -open in $X \times Y$. Hence $Gr(f)$ is a $\beta\theta$ -closed set of $X \times Y$. \square

3. A characterization of $\beta\theta$ - $T_{\frac{1}{2}}$ spaces

Lemma 3.1. Let A be a $g\beta\theta$ -closed subset of (X, τ) . Then $\beta Cl_{\theta}(A) \setminus A$ does not contain a nonempty $\beta\theta$ -closed set.

Proof. Let F be a $\beta\theta$ -closed set such that $F \subset \beta Cl_{\theta}(A) \setminus A$. Clearly $A \subset F^c$, where A is $g\beta\theta$ -closed and F^c is $\beta\theta$ -open . Thus $\beta Cl_{\theta}(A) \subset F^c$, or equivalently $F \subset (\beta Cl_{\theta}(A))^c$. Since by assumption $F \subset \beta Cl_{\theta}(A)$, then $F \subset (\beta Cl_{\theta}(A))^c \cap \beta Cl_{\theta}(A) = \emptyset$. This shows that F coincides with the void-set. \square

Definition 6. A topological space (X, τ) is $\beta\theta$ - $T_{\frac{1}{2}}$ if every $g\beta\theta$ -closed set is $\beta\theta$ -closed.

Theorem 3.2. For a topological space (X, τ) the following conditions are equivalent:

- (i) X is $\beta\theta$ - $T_{\frac{1}{2}}$,
- (ii) For each $x \in X$, $\{x\}$ is $\beta\theta$ -closed or $\beta\theta$ -open.

Proof. (i) \Rightarrow (ii): Suppose that for some $x \in X$, $\{x\}$ is not $\beta\theta$ -closed. Then $\{x\}^c$ is not $\beta\theta$ -open. Since X is the only $\beta\theta$ -open containing $\{x\}^c$, the set $\{x\}^c$ is $g\beta\theta$ -closed and so it is $\beta\theta$ -closed in the $\beta\theta$ - $T_{\frac{1}{2}}$ space (X, τ) . Therefore $\{x\}^c$ is $\beta\theta$ -closed or equivalently $\{x\}$ is $\beta\theta$ -open.

(ii) \Rightarrow (i): Let $A \subset X$ be $g\beta\theta$ -closed. Let $x \in \beta Cl_{\theta}(A)$. We will show that $x \in A$. By the hypothesis, the singleton $\{x\}$ is either $\beta\theta$ -closed or $\beta\theta$ -open. We consider these two cases.

Case 1. $\{x\}$ is $\beta\theta$ -closed: Then if $x \notin A$, there exists a $\beta\theta$ -closed set in $\beta Cl_{\theta}(A) - A$. By Lemma 3.1 $x \in A$.

Case 2. $\{x\}$ is $\beta\theta$ -open: Since $x \in \beta Cl_{\theta}(A)$, then $\{x\} \cap A \neq \emptyset$. Thus $x \in A$.

Hence in both cases, we have $x \in A$, i.e., $\beta Cl_{\theta}(A) \subset A$ or equivalently A is $\beta\theta$ -closed since the inclusion $A \subset \beta Cl_{\theta}(A)$ is trivial. \square

In the following theorem, we give a new characterization of $\beta\theta$ - $T_{\frac{1}{2}}$ spaces by using the notion of ap- $\beta\theta$ -open maps.

Theorem 3.3. Let (Y, σ) be a topological space. Then the following statements are equivalent.

- (i) (Y, σ) is a $\beta\theta$ - $T_{\frac{1}{2}}$ space,
- (ii) For every space (X, τ) and every map $f : (X, \tau) \rightarrow (Y, \sigma)$, f is ap- $\beta\theta$ -open.

Proof. (i) \rightarrow (ii): Let B be a $g\beta\theta$ -closed subset of (Y, σ) and suppose that $B \subseteq f(A)$ where $A \in \beta\theta O(X, \tau)$. Since (Y, σ) is a $\beta\theta$ - $T_{\frac{1}{2}}$ space, B is $\beta\theta$ -closed (i.e., $B = \beta Cl_{\theta}(B)$). Therefore $\beta Cl_{\theta}(B) \subseteq f(A)$. Then f is ap- $\beta\theta$ -open.

(ii) \rightarrow (i): Let B be a $g\beta\theta$ -closed subset of (Y, σ) and let X be the set Y with the topology $\tau = \{\emptyset, B, X\}$. Finally let $f : (X, \tau) \rightarrow (Y, \sigma)$ be the identity map. By assumption f is ap- $\beta\theta$ -open. Since B is $g\beta\theta$ -closed in (Y, σ) and $\beta\theta$ -open in (X, τ) and $B \subseteq f(B)$, it follows that $\beta Cl_{\theta}(B) \subseteq$

$f(B) = B$. Hence B is $\beta\theta$ -closed in (Y, σ) . Therefore (Y, σ) is a $\beta\theta$ - $T_{\frac{1}{2}}$ space. \square

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M. Caldas

Departamento de Matematica Aplicada
Universidade Federal Fluminense
Rua Mario Santos Braga ,s/n?,
CEP: 24020-140 ,
Niteroi , RJ BRASIL
e-mail: gmamccs@vm.uff.br

and

S. Jafari

College of Vestsjaelland South,
Herrestraede 11,
4200 Slagelse,
DENMARK
e-mail: jafari@stofanet.dk