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Complementary nil vertex edge dominating sets

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Abstract

Dominating sets play a vital role in day-to-day life problems. For providing effective services in a location, central points are to be identified. This can easily be achieved by graph theoretic techniques. Such graphs and relevant parameters are introduced and extensively studied. One such parameter is complementary nil vertex edge dominating set(cnved-set). A vertex edge dominating set(ved-set) of a connected graph G with vertex set V is said to be a complementary nil vertex edge dominating set(cnved-Set) of G if and only if $V-D$ is not a ved-set of G. A cnved-set of minimum cardinality is called a minimum cnved-set(mcnved-set)of G and this minimum cardinality is called the complementary nil vertex-edge domination number of G and is denoted by $\gamma_{cnve}(G)$. We have given a characterization result for a ved-set to be a cnved-set and also bounds for this parameter are obtained.

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1. Introduction & Preliminaries

Domination is an active topic in graph theory and has numerous applications to distributed computing, the web graph and adhoc networks. Haynes et al.[3] gave a comprehensive introduction to the theoretical and applied facets of domination in graphs.

A subset S of the vertex set V of G is said to be a dominating set of G if each vertex in $V - D$ is adjacent to some vertex of D. The domination number $\gamma(G)$ is the minimum cardinality of the dominating set of G[3]. A subset $E^{'}$ of the edge set E of a graph G is said to be an edge dominating set of G if each edge in $E - E^{'}$ is adjacent to some edge in $E^{'}$. The edge domination number $\gamma^{'}(G)$ is the minimum cardinality of the edge dominating set of G[3]. A subset D of vertices is said to be a vertex edge dominating set of G if each edge in G has either one of its ends from D (or) one of its ends is adjacent to a vertex in D. The vertex edge domination number $\gamma_{ve}(G)$ is the minimum cardinality of the vertex edge dominating set of G[5]. A subset D of vertices is said to be a complementary nil dominating set of G if $V - D$ is not a dominating set of G. The minimum cardinality of a complementary nil dominating set is called a complementary nil domination number of G and is denoted by $\gamma_{cnd}(G)$[6].

Many variants of vertex - vertex dominating sets have been studied. In the present paper, we introduce a new variant of vertex-edge dominating set namely complementary nil vertex edge dominating set. We have given the characterization result for vertex edge dominating set to be complementary nil vertex edge dominating set and characterized the graphs of order n having cnved number n, characterized trees of order n having cnved numbers $n-1, n-2$. Bounds for this parameter are also obtained.

All graphs considered in this paper are simple, finite, undirected and connected. For standard terminology and notation, we refer Bondy & Murthy[1].

2. Characterization and other relevant results

In this section, we mainly obtain characterization result for a proper subset of the vertex set of G to be a cnved-set of G.

Theorem 2.1. A ved - set of a (connected)graph G is a cnved-set iff there is an edge $e_1 \in F = \{e = uv \in E(G) : u, v \in D\}$ with $N[e_1] \subseteq F$.

Proof: Let D be a cnved-set of G. So $V - D$ is not a ved-set of G. So there is an edge, say e_1 with ends u_1, v_1 of G such that $u_1, v_1 \notin V - D$

and $N(u_1)\bigcap(V-D) = \phi = N(v_1)\bigcap(V-D)$. $\Rightarrow u_1, v_1 \in D, N(u_1) \subseteq D \& N(v_1) \subseteq D \Rightarrow N[e_1] \subseteq F$. Conversely assume that there is an $e = u_1v_1 \in F$ such that $N[e] \subseteq F \Rightarrow u_1, v_1 \in D \& N(u_1), N(v_1) \subseteq D$. So there is no w_1 in $V-D$ such that atleast one of u_1, v_1 is adjacent with w_1 in G. Hence $V-D$ is not a ved-set of G. Thus D is a cnved-set of G.

Note: For a cnved-set there is an edge of G whose ends are in cnved-set. So cnved- set is not an independent set in G.

Corollary 2.2. Let D be a cnved-set of a (connected)graph G. Then D has atleast two enclaves in D.

Proof: By the above Characterization Result, there is atleast one edge , say $e_1 = u_1v_1$ in $F = \{e = uv \in E(G) : u, v \in D\}$ with $N(e_1) \subseteq F \Rightarrow N(u_1) \subseteq D \& N(v_1) \subseteq D$. Hence $u_1 \& v_1$ are enclaves in D.

Result 2.3. If D_1, D_2 are ved - sets such that atleast one of them is a cnved-set then $D_1 \bigcap D_2 \neq \phi$.

Proof: If $D_1 \bigcap D_2 = \phi$ then $D_1 \subseteq V - D_2$ and $D_2 \subseteq V - D_1 \Rightarrow$ Both D_1, D_2 are not cnved-sets and this contradicts the hypothesis.

We now give the bounds for cnved number of connected graphs.

Theorem 2.4 If G is a connected graph , then

$$\gamma_{ve}(G) + 2 \leq \gamma_{cnve}(G)$$

Proof: Let D be a minimum cnved-set of G. By the Characterization Result there is an edge $e_1 = u_1v_1$ in F such that $N[e_1] \subseteq F$. Hence follows that $D - \{u_1, v_1\}$ is a ved-set of G. Thus $\gamma_{ve}(G) \leq |D| - 2 = \gamma_{cnve}(G) - 2$, which implies $\gamma_{ve}(G) + 2 \leq \gamma_{cnve}(G)$.

Furthermore the lower bound is attained in the case of P_8. Hence the bound is sharp.

A lower bound for $\gamma_{cnve}(G)$ is obtained in terms of the number of edges(ϵ) and maximum degree $\Delta(G)$ of the vertices of G.

Corollary 2.5. For any graph $G(\neq K_2)$, $\lceil \frac{\epsilon}{\Delta(G)(\Delta(G)-1)} \rceil + 2 \leq \gamma_{cnve}(G)$ (ϵ being the number of edges of G).

Proof: Clearly $\Delta(G) \geq 2$. Any vertex v can have atmost $\Delta(G)$ neighbours. Furthermore any neighbour u of v can dominate atmost $\Delta(G) - 1$ edges(excluding the edge uv)

$$\Rightarrow \lceil \frac{\epsilon}{\Delta(G)(\Delta(G)-1)} \rceil \leq \gamma_{ve}(G).$$

Then by the above Theorem the inequality follows.

Note: The bound is sharp as it is attained in the case of C_4. For any k - regular graph $G(\neq K_2)$ with n vertices ,

$$\lceil \frac{n}{k-1} \rceil + 2 \leq \gamma_{cnve}(G)$$

Proof: The proof follows from the above theorem and the fact that

$k \geq 2, \Delta(G) = k, \epsilon = nk.$

Theorem 2.7. If G is a connected graph of order n and having ϵ edges, then

$$\lceil \frac{2\epsilon - n^2 + 5n}{4} \rceil \leq \gamma_{cnve}(G).$$

Proof: Let S be a minimum cnved-set of G. Since $V - S$ is not a cnved-set of G, there is an edge uv such that $N[uv] \subseteq S$. Each of u, v are non adjacent to all the vertices in $V - S$. So

$$\epsilon \leq n_{C_2} - 2(n - \gamma_{cnve}(G))$$

$\Rightarrow \lceil \frac{2\epsilon - n^2 + 5n}{4} \rceil \leq \gamma_{cnve}(G)$

Note: The bound is sharp as it is attained in the case of $< v_4v_1v_2v_3v_1 >$.

Theorem 2.8 For a connected graph G with $g(G) \neq 3$, $2\delta(G) \leq \gamma_{cnve}(G)$.

Proof: Let S be a minimum cnved-set of G. Since $V-S$ is not a cnved-set of G, there is an edge $e=uv$ such that $N[e]\subseteq S$. That is $|N[e]|\leq|S|$. Hence $2\delta(G)\leq\gamma_{cnve}(G)$.

Note: The bound is sharp as it is attained in the case of C_5.
Corollary 2.9 For a connected Bipartite graph G , $2\delta(G)\leq\gamma_{cnve}(G)$.

Proof: The proof follows from the fact that G cannot have odd cycles.

Note The bound is sharp as it is attained in the case of C_4.

Theorem 2.10. If G is a connected graph , then

$$\gamma_{cnve}(G)\leq\gamma_{ve}(G)+2\Delta(G).$$

Proof: Let D be the minimum vertex edge dominating set for G. Then for any edge e in G, $D\bigcup N[e]$ is a cnved-set of G. Hence

$$\gamma_{cnve}(G) \quad\leq\quad |D\bigcup N[e]|$$

$$\leq|D|+d(u)+d(v)$$
$$\leq\gamma_{ve}(G)+2\Delta(G)$$

Note: The bound is sharp as it is attained in the case of C_6.

Remark: Since $\gamma_{ve}(G)\leq\gamma(G)$, follows that $\gamma_{cnve}(G)\leq\gamma(G)+2\Delta(G)$

Theorem 2.11. If G is a connected graph having a pendant vertex, then $\gamma_{cnve}(G)\leq\gamma_{ve}(G)+\Delta(G)$.

Proof: Suppose that D is a minimum vertex edge dominating set for G. Let v be adjacent to pendant vertex u(say) in G. Clearly $D\bigcup N[v]$ is a cnved-set for G. Hence

$$\gamma_{cnve}(G)\leq|D\bigcup N[v]|$$

$\leq \gamma_{ve}(G) + \Delta(G)$

Note: The bound is sharp as it is attained in the case of P_5.

Corollary 2.12. If $G = P_n$, then $\gamma_{cnve}(G) \leq \gamma_{ve}(G) + 2$.

Proof: Since $\Delta(G) = 2$, the proof follows from the above theorem.

Theorem 2.13. If G is a connected graph, then

$$\gamma_{cnve}(G) \leq \gamma^{'}(G) + 2\Delta(G) - 1.$$

Proof: Let $E^{'} = \{e_1 = x_1y_1, e_2 = x_2y_2, ..., e_{\gamma^{'}(G)} = x_{\gamma^{'}(G)}y_{\gamma^{'}(G)}\}$ be a minimum edge dominating set for G. Let $e_i(1 \leq i \leq \gamma^{'}(G))$ be any edge in $E^{'}$. Then $\{x_1, x_2, x_3..., x_{\gamma^{'}(G)}, N(e_i)\}$ is a cnved-set for G whose cardinality is $\gamma^{'}(G) + d(e^{'}) - 1$. Hence

$$\gamma_{cnve}(G) \leq \gamma^{'}(G) + d(x_i) + d(y_i) - 1$$

$\leq \gamma^{'}(G) + 2\Delta(G) - 1$

Note: The bound is sharp as it is attained in the case of K_2.

Theorem 2.14. If G is a tree with n vertices and $diam(G) \geq 3$, then $\gamma_{cnve}(G) < n - \nu^{'} + \Delta(G)$,where $\nu^{'}$ is the number of pendant vertices in G.

Proof: Clearly the non pendant vertices of G along with all the neighbours of a support vertex $v(say)$ forms a cnved - set for G, whose cardinality is $n - \nu^{'} + d(v)$. Hence the inequality follows.

Theorem 2.15. If G is a connected graph such that $\overline{G}$(the complement of G) is connected, then $\gamma_{cnve}(G) + \gamma_{cnve}(\overline{G}) \leq \frac{5n}{2} + 2(\Delta(G) - \delta(G))$.

Proof: By the Remark in Theorem.2.10,

$\gamma_{cnve}(G) \leq \gamma(G) + 2\Delta(G)$

$\gamma_{cnve}(\overline{G}) \leq \gamma(\overline{G}) + 2\Delta(\overline{G})$

So,

$$\begin{aligned}\gamma_{cnve}(G)+\gamma_{cnve}(\overline{G}) &\leq \gamma(G)+\gamma(\overline{G})+2(\Delta(G)+\Delta(\overline{G}))\\ &\leq \frac{n}{2}+2+2(\Delta(G)+n-\delta(G)-1)(see[3])\\ &\leq \frac{5n}{2}+2(\Delta(G)-\delta(G))\end{aligned}$$

Theorem 2.16. If G is a connected graph with $\gamma(G) > 3$, then $\gamma_{cnve}(G)+\gamma_{cnve}(\overline{G}) \leq 3(n-\delta(G))+2\Delta(G)$.

Proof: By the Remark in Theorem.2.10 and [2] the proof follows.

Theorem 2.17. If G is a connected graph with $\gamma(\overline{G}) > 3$, then

$\gamma_{cnve}(G)+\gamma_{cnve}(\overline{G}) \leq 2\Delta(G)-\delta(G)+3$.

Proof: By the Remark in Theorem.2.10 and [2] the proof follows.

Theorem 2.18. If G is a connected graph such that $\overline{G}$ is connected, then $\gamma_{cnve}(G)+\gamma_{cnve}(\overline{G}) \leq (n-1)(n-2)+2$.

Proof: For any graph G,

$$\begin{aligned}\gamma_{cnve}(G) &\leq n\\ &= 2(n-1)-n+2\end{aligned}$$

$\leq 2\epsilon-n+2$

Similarly, $\gamma_{cnve}(\overline{G}) \leq 2\epsilon^{'}-n+2$(here $\epsilon^{'}$ is the number of edges in $\overline{G}$). So,

$$\gamma_{cnve}(G)+\gamma_{cnve}(\overline{G}) \leq 2(\epsilon+\epsilon^{'})-2(n-2)$$

$\leq n(n-1)-2(n-2)$

=(n -1)(n - 2)+ 2

Note: If G is a connected graph with n vertices, then $\gamma_{cnved}(G) \leq n$.

Now, we characterize the graphs for which $\gamma_{cnve}(G) = n$.

Theorem 2.19. For a connected graph G with n vertices, $\gamma_{cnved}(G) = n$ iff for each edge v_1v_2 in G, $N(v_1) \bigcup N(v_2) = V$.

Proof: Assume that $\gamma_{cnve}(G) = n$. Suppose that there is an edge v_1v_2 in G such that $N(v_1) \bigcup N(v_2) \neq V$. Consider the set $V - [N(v_1) \bigcup N(v_2)]$. Let $E^{'} = \{uv : u, v \in V - [N(v_1) \bigcup N(v_2)]\}$.

If $E^{'} = \phi$,then $[N(v_1) \bigcup N(v_2)](\subset V)$ is a cnved-set of G whose cardinality is less than n.
Suppose $E^{'} \neq \phi$.

Let $E^{'} = \{x_1y_1, x_2y_2, ..., x_sy_s\}$. Then $[N(v_1) \bigcup N(v_2)] \bigcup \{x_1, x_2, ..., x_s\}$ is a cnved-set of G whose cardinality is less than n.

Hence in either case we get a contradiction to our assumption.

Assume that the converse holds. Let D be the minimum cnved-set of G. By the Characterization Result for cnved-set there is an edge v_1v_2 such that the neighbours of v_1, v_2 are from D. Then by our assumption $N(v_1) \bigcup N(v_2) = V$. Hence $\gamma_{cnve}(G) = n$.

Corollary 2.20. If $G = C_n$, then $\gamma_{cnve}(G) = n$ iff $n = 3, 4$.

Corollary 2.21. If $G = P_n$, then $\gamma_{cnve}(G) = n$ iff $n = 2, 3$.

Corollary 2.22.

1. $\gamma_{cnve}(S_n) = n, n \geq 3$
2. $\gamma_{cnve}(K_n) = n, n \geq 3$
3. $\gamma_{cnve}(K_{m,n}) = m + n$

Theorem 2.23. Let G be a connected graph with $diam(G) = 2$ and circumference of G is three, then $\gamma_{cnve}(G) \neq n$ iff there is a triangle T in G for which there is a vertex in $V - V(T)$ adjacent to exactly one vertex in T.

Proof: Assume that $\gamma_{cnve}(G) \neq n$.

By Theorem.2.19. there is an edge uv in G such that $N[u]\bigcup N[v] \neq V$. Since $diam(G) = 2$ and $c(G) = 3$, for $x \in V - [N[u]\bigcup N[v]]$ there is $w \in N[u]\bigcap N[v]$ such that wx is an edge in G. So $< uvw > (= T)$ is a triangle in G. Supposing that all the vertices in $V - V(T)$ are adjacent to atleast two vertices in T we get a contradiction to that $N[u]\bigcup N[v] \neq V$. Hence there is atleast one vertex in $V - V(T)$ which is adjacent to exactly one vertex in T.

The converse part is clear.

Theorem 2.24. If G is a connected graph such that $\overline{G}$ is connected and $diam(G) \geq 4$, then $\gamma_{cnve}(G) + \gamma_{cnve}(\overline{G}) \leq (n-1)(n-2)$.

Proof: By Theorem.2.19, observe that for any graph G,

$$\gamma_{cnve}(G) \quad \leq \quad n-1$$

Now we construct the proof as in the case of Theorem.2.18.

Corollary 2.25. G be a tree with n vertices, then $\gamma_{cnve}(G) = n$ iff $G \cong S_n$.

Proof: Assume that $\gamma_{cnve}(G) = n$. Then by the Characterization Result for each edge v_1v_2 in G, $N(v_1)\bigcup N(v_2) = V$. So for any pendant edge uv in G, $N(u)\bigcup N(v) = V$, which implies one of u, v is adjacent to all the vertices in G. W.l.g assume that v is the vertex adjacent to all the vertices in G(i.e u is a pendant vertex). Since G is a tree there is no edge between $v_1, v_2 \in V - \{v\}$(i.e. all the vertices in $V - \{v\}$ are pendant). Hence $G \cong S_n$.

For the converse part, any edge uv in S_n has the property that $N(u)\bigcup N(v) = V$. Hence by the Characterization Result the claimant holds.

Now we characterize the graphs for which $\gamma_{cnve}(G) = 3$.

Theorem 2.26. G be a tree, then $\gamma_{cnve}(G) = 3$ iff $G = P_3$ or G is obtained by adding zero or more leaves to exactly one pendant vertex of P_4.

Proof: Assume that $\gamma_{cnve}(G) = 3$. Let $D = \{v_1, v_2, v_3\}$ be a minimum cnved-set of G. Then by the Characterization Result $< D >$ is connected .

Clearly atleast one of v_1 or v_3 is a pendant vertex.

Suppose both v_1, v_3 are pendant vertices.

If there is any vertex in G adjacent to v_2 then it should be a member of D, which is a contradiction to our assumption. Hence $G = P_3$.
Suppose exactly one of v_1, v_3 is a pendant vertex.

W.l.g assume that v_1 is the pendant vertex. Clearly no vertex other than v_1, v_3 can be adjacent with v_2. Hence any vertex or edge in $< V - \{v_1, v_2, v_3\} >$ is adjacent to v_3. Then $G = P_4$ or G is obtained by adding zero or more leaves to exactly one pendant vertex of P_4.

The converse part is clear.

Theorem 2.27. *G be a connected graph with $\delta(G) \geq 2$. Then $\gamma_{cnve}(G) = 3$ iff*

(i) there is a C_3 edge disjoint with the other cycles in G.

(ii) each edge in G lies on a cycle of length atmost four that has a common vertex with C_3.

Proof: Assume that $\gamma_{cnve}(G) = 3$.
Let D be a minimum cnved- set of G. By the Characterization Result and by the hypothesis,

$< D > = \ C_3 \ = < uvw > (say)$. By our assumption none of the edges of C_3 are common to two cycles. Hence C_3 is edge disjoint with the remaining cycles in G. If any edge in G lies on a cycle of length greater than four then there is atleast one edge not dominated by the vertices of D. Hence (ii) holds.

Conversely let C_3 be the cycle through u, v, w satisfying conditions $(i)\&(ii)$.

Denote $D = \{u, v, w\}$. Let v be the vertex in C_3 through which the vertices in $V - V(C_3)$ reach the vertices $\{u, w\}$. Let v_1v_2 be an any edge in G. Then by (ii) of our assumption, v_1v_2 lies on a cycle of length three or four for which v is one of the vertices. In either case v_1v_2 is ve dominated by v. Hence D is a cnved-set. Since $\delta(G) \geq 2$, D is the $\gamma_{cnve}(G) - set$ of cardinality 3.

This proves the result.

G is a union of edge disjoint triangles, where all the triangles having a common vertex, then $\gamma_{cnve}(G) = 3$.

Theorem 2.29. G be a tree and D be the set of all pendant vertices in G. Then D is a cnved-set iff $G = P_2$.

Proof: Suppose that D is cnved-set of G. Then by the Characterization Result for cnved-set, there is an edge $v_1v_2(v_1, v_2 \in D)$ in G all of whose neighbours are from D. This implies v_1, v_2 are pendant vertices. Hence $G = P_2$.

The converse part is clear.

Theorem 2.30. G be a tree with n vertices, then $\gamma_{cnve}(G) = n - 2$ iff $G = P_5$ or $G = P_6$ or $G = S_{1,2}$ or $G = S_{2,2}$.

Proof: Assume that $\gamma_{cnve}(G) = n - 2$.

Suppose $diam(G) = k$ where $k \geq 6$.

If any two pendant vertices v_1, v_2 in G are adjacent(with v_3), then $V - \{v_1, v_2, v_3\}$ is a cnved-set of cardinality $n - 3$ which is a contradiction to our assumption.
Suppose that $P = < v_1v_2v_3.....v_{k-1}v_k >$ be the diammetral path in G. Then $V - \{v_1, v_2, v_{k-1}, v_k\}$ is a cnved-set of G of cardinality less than $n-2$ leading to a contradiction.

Hence $diam(G) < 6$.

Suppose $diam(G) = 5$.

If G has exactly two pendant vertices, then $G = P_6$,which implies $\gamma_{cnve}(G) = n-2$. Suppose G has more than two pendant vertices. Clearly no two pendant vertices in G have a common neighbour.
Let $P = < v_1v_2v_3v_4v_5v_6 >$ be the diammetral path in G. Any vertex in $V - \{v_1, v_2, v_3, v_4, v_5, v_6\}$ can be adjacent to v_3 or v_4. If v_7 is a pendant vertex not adjacent to v_3 or v_4 then there is a path from v_1 to v_7 through v_3 or a path from v_6 to v_7 through v_4. In any case $V - \{v_1, v_2, v_5, v_6\}$ is a cnved-set of cardinality less than $n - 2$ which contradicts our assumption. If each vertex in $V - \{v_1, v_2, v_3, v_4, v_5, v_6\}$ is adjacent to v_3 or v_4, then $V - \{v_1, v_5, v_6\}$ is a cnved-set of cardinality less than $n-2$ which is a contradiction.

Hence when $diam(G) = 5$, $G = P_6$.

Suppose $diam(G) = 4$.

If $G = P_5$, then $\gamma_{cnve}(G) = n - 2$. Suppose $G \neq P_5$. Clearly no two pendant vertices are adjacent. Let $P = < v_1v_2v_3v_4v_5 >$ be the diammetral path in G. Then any vertex in $V - \{v_1, v_2, v_3, v_4, v_5\}$ is adjacent to v_3 or a pendant vertex at a distance two from v_3. In the later case $V - \{v_1, v_2, v_4, v_5\}$ is a cnved-set of cardinality $n - 4$ which contradicts our assumption. In the former case $V - \{v_1, v_2, v_3\}$ is cnved-set of cardinality $n - 3$ which again contradicts our assumption.

Hence when $diam(G) = 4$, $G = P_5$.

Suppose $diam(G) = 3$.

If $G = P_4$, then $\gamma_{cnve}(G) = n - 1 > n - 2$, which is a contradiction. Suppose $G \neq P_4$. Then $G \cong S_{p,q}(p + q \geq 3)$. If $max\{p, q\} \geq 3$, then $\gamma_{cnve}(G) = n - max\{p, q\} < n - 2$, which is a contradiction. Therefore $max\{p, q\} = 2$. Then $G = S_{1,2}$ or $G = S_{2,2}$. In any case $\gamma_{cnve}(G) = n - 2$.

Hence when $diam(G) = 4$,$G = S_{1,2}$ or $G = S_{2,2}$.

If $diam(G) \leq 2$, then by Theorem.2.19. $\gamma_{cnve}(G) = n$.
The converse part is clear.

Theorem 2.31. G be a tree with n vertices, then $\gamma_{cnve}(G) = n - 1$ iff $G = P_4$.

Proof: Suppose that $\gamma_{cnve}(G) = n - 1$.

Suppose $diam(G) \geq 6$.
Let $P =< v_1v_2v_3v_4....v_k > (k \geq 7)$ be a diammetral path in G. Then $V - \{v_1, v_k\}$ is a cnved-set of G

$$\Rightarrow \gamma_{cnve}(G) \leq n - 2 < n - 1, \text{ a contradiction.}$$

Hence $diam(G) < 6$.

Suppose $diam(G) = 5$.
If $G = P_6$, then $\gamma_{cnve}(G) = n - 2$. So $G \neq P_6$. Clearly any two pendant vertices in G do not have a common neighbour. Let v be an arbitrary vertex in $V - \{v_1, v_2, v_3, v_4, v_5, v_6\}$. Then
(i) v can be adjacent to v_3 or v_4
(ii) v is at a distance three from v_3 or v_4.

In any of the cases, $\gamma_{cnve}(G) = n - 2 < n - 1$.
Suppose $diam(G) = 4$.

If $G = P_5$, then $\gamma_{cnve}(G) = n-2$. So $G \neq P_5$. Clearly any two pendant vertices in G do not have a common neighbour. Let v be an arbitrary vertex in $V - \{v_1, v_2, v_3, v_4, v_5\}$. Then v is adjacent to v_3 or at a distance two from v_3. In any case $\gamma_{cnve}(G) = n - 2 < n - 1$.
Suppose $diam(G) = 3$.

If $G = P_4$, then $\gamma_{cnve}(G) = n - 1$. Suppose $G \neq P_4$. This implies there is a pair of pendant vertices in G having a common neighbour. Then $\gamma_{cnve}(G) < n - 1$.
If $diam(G) \leq 2$, then $\gamma_{cnve}(G) = n > n - 1$.
The converse part is clear.

Proposition 2.31.

1. $\gamma_{cnve}(S_{n,m}) = min\{n + 1, m + 1\}$
2. $\gamma_{cnve}(C_n) = n, n \geq 3$
3. $\gamma_{cnve}(P_n) = n$

Result 2.33. G be connected graph and D be a cnved-set of G. If $E^{'} = \{uv/u \in D \ or \ v \in D\}$, then the minimum edge domination number is atmost $|E^{'}| - 1$.

Proof: Assume that the hypothesis holds. Clearly $E^{'}$ is an edge dominating set of G. Since D is a cnved-set of G there is v_1v_2 in $E(G)$ such that $N[v_1] \subseteq D; N[v_2] \subseteq D \Rightarrow v_1v_2 \in E^{'}$. Clearly $E^{'} - \{v_1v_2\}$ is also an edge dominating set of G. Hence min edge domination number is atleast $|E^{'}| - 1$.

Result 2.34. G be a connected graph and D be a cnved-set. Then D is connected ved-set of G iff $< D >$ has exactly one component and $|D| \geq 3$.

Proof: Suppose that the hypothesis holds. Assume that D is connected ved-set of G. Then $< D >$ has exactly one component. By the nature of D, it has atleast three vertices. Converse is clear.

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