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On (γ, δ)-Bitopological semi-closed set via topological ideal

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Abstract

In this paper we introduce a new class of generalized closed sets in bitopological space using local function, two extension operators and semi-open sets. We have also investigated some properties in subspace bitopology defining kernel and image.

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1. Introduction

Levine [2], introduced the concept of generalized closed set (briefly g-closed set) in 1970. An ideal I of a topological space (X, τ) is a non-empty collection of subsets satisfying the following two properties:

1)$A \in I$ and $B \subset A$ implies $B \in I$.

2) $A \in I$ and $B \in I$ implies $A \cup B \in I$.

The notions of ideals have been applied in different branches of mathematics. In sequence spaces ideals of natural numbers have been considered and different types of ideal convergent(I-convergent) sequence spaces have been introduced and their algebraic and topological properties have been investigated by Tripathy and Hazarika ([8], [9]), Tripathy and Mahanta [10], Tripathy, Sen and Nath [11] and many others.

Kuratowski [3] introduced the notion of local function of $A \subseteq X$ with respect to I and τ (briefly A^*). Let $A \subseteq X$, then $A^*(I) = \{x \in X | U \cap A \notin I$, for every open neighbourhood U of $x\}$.

Jankovic and Hamlett [4] introduced τ^*-closed set by $A \subset (X, \tau, I)$ is called τ^*-closed if $A^* \subseteq A$.

It is well known that $cl^*(A) = A^* \cup A$, defines a Kuratowski closure operator for a topology $\tau^*(I)$ finer than τ.

An operator γ (see for instance Kasahara [5], Ogata [6]) on a given topological space (X, τ) is a function from the topology τ into the power set $P(X)$ of X such that $V \subseteq V^\gamma$ for each $V \in \tau$, where V^γ denotes the value of γ at V.

Tong [7] called the γ-operator as an expansion. The following operators are examples of the operator γ : the closure operator $\gamma_{cl}(U) = cl(U)$, the identity operator $\gamma_{id}(U) = U$ and the interior closure operator $\gamma_{ic}(U) = int(cl(U))$. Another example of the operator γ is the γ_f- operator is defined by $U^{\gamma_f} = X \backslash Fr(U)$ where $Fr(U)$ denotes the frontier of U.

Two operators γ_1 and γ_2 are called mutually dual (Tong [7]) if $U^{\gamma_1} \cap U^{\gamma_2} = U$ for each $U \in \tau$. For example identity operator is mutually dual to any other operator while the γ_f-operator is mutually dual to the closure operator.

Dontchev et al. [1], introduced the concept of (I,γ)- generalized closed set and investigated their properties.

Definition1.1. A subset A of a topological space (X,τ) is called :

(1) semi-open if $A \subseteq cl(intA)$ and semi-closed if $int(clA) \subseteq A$.

(2) generalized closed set (see for instance [2])(briefly g-closed set) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open, where $cl(A)$ denotes the closure of A.

(3) (I,γ)-generalized closed set (Dontchev etal.[1]) (briefly (I,γ)-g-closed set) if $A^* \subseteq U^\gamma$ whenever $A \subseteq U$ and U is open in (X,τ).

(4) A subset of a topological space is clopen if it is both closed and open set.

2. Some properties of (γ,δ)-BSC-sets

At first we define some existing definitions in topological space in terms of bitopological space. Let (X,τ_1,τ_2) be a bitopological space. We define γ-operator in a bitopological space as τ_i-γ:$\tau_i \to P(X)$ such that τ_i-$U \subseteq \tau_i$-U^γ whenever $i = 1,2$.

This may be extended to semi-open sets such that τ_i^s-γ: $\tau_i^s \to P(X)$ such that τ_i^s-$U \subseteq \tau_i^s$-$U^\gamma (= U^{\tau_i^s - \gamma})$ whenever $i = 1,2$. Here τ_i^s-U indicates a semiopen set U of τ_i

Now we redefine local function on (X,τ_1,τ_2) with respect to an ideal I on X by $A^*(I,\tau_i)$ or τ_i-A^* $=\{x \in X | U \cap A \notin I$ for every $U \in \tau_i, x \in U\}$ and τ_i^s-$A^*$$=\{x \in X | U \cap A \notin I$ for every $U \in \tau_i^s, x \in U\}$.

A is called τ_i^*-closed set if τ_i-$A^* \subseteq A$ and A is called τ_i^{s*}-closed set if

τ_i^s-$A^* \subseteq A$

τ_i-$cl^*(A) = A \cup \tau_i$-A^* and τ_i^s-$cl^*(A) = A \cup \tau_i^s$-$A^*$.

We define ideal in product bitopological space by if I_1 and I_2 are ideals of two bitopological space. We define $I_1 \times I_2$=$\{A \times B | A \in I_1, B \in I_2\}$, Then clearly $I_1 \times I_2$ is an ideal of product bitopological space.

So, we define $(\tau\theta)_i$-$(A \times B)^*$=$\{(x, y) | (U \times V) \cap (A \times B)$ is not subset of $P_1 \times P_2 \in I_1 \times I_2, x \in U, y \in V; U \in \tau_i, V \in \theta_i\}$ for $i = 1, 2$.

Throughout this paper BS denotes the word bitopological space.

Definition 2.1. Let $(X, \tau_1, \tau_2, I, \gamma, \delta)$ be a BS, $A \subseteq X$ is said to be (γ, δ)-BSC-set if τ_2^s-$U^\gamma \subseteq \tau_1$-$A^* \subseteq \tau_2^s$-$U^\delta$ whenever $A \subseteq \tau_2^s$-U where γ, δ are two expansion operators as defined above.

$(X, \tau_1, \tau_2, I_1, \gamma, \delta)$ and $(Y, \theta_1, \theta_2, I_2, \zeta, \eta)$ be two bitopological spaces then $(A \times B) \subseteq (X \times Y)$ is said to be *Product*-2-(γ, δ)-BSC-set if τ_2^s-$U^\gamma \times \theta_2^s$-$V^\zeta \subseteq (\tau\theta)_1$-$(A \times B)^* \subseteq \tau_2^s$-$U^\delta \times \theta_2^s$-$V^\eta$, whenever $(A \times B) \subseteq (\tau_2^s$-$U \times \theta_2^s$-$V)$.

Let $(X, \tau_1, \tau_2, I, \gamma, \delta)$ be a bitopological space and $A \subseteq X$ is called (γ, δ)-BSO-set if $(X \setminus A)$ is (γ, δ)-BSC-set.

The collection of all (γ, δ)-BSC-set of $(X, \tau_1, \tau_2, I, \gamma, \delta)$ is denoted by $BSC(X)$ and (γ, δ)-BSO-set is denoted by $BSO(X)$.

Example 2.1. Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$, $\tau_2 = \{\emptyset, X, \{b\}\}$, I=$\{\emptyset, \{c\}\}$. We define $U^{\tau_2^s - \gamma} = \tau_2^s$-$U$ and $U^{\tau_2^s - \delta} = \tau_2$-$cl(\tau_2^s$-$U)$. Let us consider $A = \{a, b\}$ then τ_2-semi-open set containing A is X. Then τ_1-$A^* = X$, $U^{\tau_2^s - \gamma} = X$, $U^{\tau_2^s - \delta} = X$. Thus $A = \{a, b\}$ is a $(\gamma_{id}, \delta_{cl})$-$BSC$-set. Hence $\{c\}$ is $(\gamma_{id}, \delta_{cl})$-$BSO$-set

Theorem 2.1. Arbitrary union of (γ, δ)-BSC-sets is (γ, δ)-BSC-set.

Proof. Let $\cup_{i \in I} A_i \subseteq \tau_2^s$-$U$. Then $A_i \subseteq \tau_2^s$-U for all $i \in I$. As A_i is (γ, δ)-BSC-set then τ_2^s-$U^\gamma \subseteq \tau_1$-$A_i^* \subseteq \tau_2^s$-$U^\delta$. Thus τ_2^s-$U^\gamma \subseteq \cup_{i \in I}(\tau_1$-$A_i^*) \subseteq \tau_2^s$-$U^\delta$ which implies τ_2^s-$U^\gamma \subseteq \tau_1$-$(\cup_{i \in I} A_i)^* \subseteq \tau_2^s$-$U^\delta$. Thus $\cup_{i \in I} A_i$ is (γ, δ)-BSC-set.

From above result it can be easily proved, if $(X,\tau_1,\tau_2,I,\gamma,\delta)$ be a bitopological space then arbitrary intersection of locally finite family of (γ,δ)-BSO-sets is (γ,δ)-BSO-set.

Theorem 2.2. A subset of a (γ,δ)-BSC-set is not necessarily a (γ,δ)-BSC-set.

Proof. The result can be verified on considering the following example.

Example 2.2. Let $X=\{a,b,c\}$, $\tau_1=\{\emptyset,X,\{a\},\{b\},\{a,b\}\}$, $\tau_2=\{\emptyset,X,\{b\}\}$, I=$\{\emptyset,\{c\}\}$. We define $U^{\tau_2^s-\gamma}=\tau_2^s$-$U$ and $U^{\tau_2^s-\delta}=\tau_2$-$cl(\tau_2^s$-$U)$. Let $A=X, B=\{a,b\}, C=\{a\}$; then A,B are $(\gamma_{id},\delta_{cl})$-$BSC$-set. Then τ_2 semi-open sets containing C is $X,\{a,b\}$. Here τ_1-$C^*=\{a,c\}$. It can verified that C is not $(\gamma_{id},\delta_{cl})$-$BSC$-set.

Note 2.1. The above theorem implies that superset of a (γ,δ)-BSO-set may not be (γ,δ)-BSO-set.

Theorem 2.3. A superset of a (γ,δ)-BSC-set is a (γ,δ)-BSC-set.

Proof. Let $A\subseteq B$ and A is (γ,δ)-BSC set. Let $B\subseteq U$ and if possible let B be not a (γ,δ)-BSC-set. Then obviously τ_2^s-$U^\gamma\subseteq\tau_1$-$A^*\subseteq\tau_1$-B^* but as B is not a (γ,δ)-BSC-set; only criteria is τ_1-B^* is not a subset of τ_2^s-U^δ. This implies A is not (γ,δ)-BSC-set; but this is a contradiction. Thus B is a (γ,δ)-BSC-set.

Note 2.2. The above theorem implies that a subset of a (γ,δ)-BSO-set is (γ,δ)-BSO-set.

Remark 2.1. $\emptyset$ is not a (γ,δ)-BSC set but X is not necessarily a (γ,δ)-BSC set.

Proof. For any ideal I, always $\emptyset\in I$, then τ_1-$\emptyset^*=\emptyset$. Let $\emptyset\neq\tau_2^s$-U. If possible let $\emptyset$ be (γ,δ)-BSC-set then $\emptyset\subseteq\tau_2^s$-$U\subseteq\tau_2^s$-$U^\gamma\subseteq\emptyset$ whenever $\emptyset\subseteq\tau_2^s$-$U$. It implies τ_2^s-$U=\emptyset$; a contradiction. Thus $\emptyset$ is not a (γ,δ)-BSC set. The next part follows from the following example

Example 2.3. Let us consider $X=\{a,b,c\}$, $\tau_1=\{\emptyset,X,\{a\},\{b\},\{a,b\}\}$,

$\tau_2 = \{\emptyset, X, \{b\}\}$, I=$\{\emptyset, \{c\}\}$. We define $U^{\tau_2^s - \gamma} = \tau_2^s$-$U$ and $U^{\tau_2^s - \delta} = \tau_2$-$cl(\tau_2^s$-$U)$; then X is $(\gamma_{id}, \delta_{cl})$-$BSC$-closed set.

Next we consider $X = \{a, b\}$, $\tau_1 = \{\emptyset, X, \{a\}\}$, $\tau_2 = \{\emptyset, X, \{b\}\}$ and I=$\{\emptyset, \{a\}\}$. Define $U^{\tau_2^s - \gamma} = \tau_2^s$-$U$ and $U^{\tau_2^s - \delta} = \tau_2$-$cl(\tau_2^s$-$U)$; then X is not $(\gamma_{id}, \delta_{cl})$-$BSC$-closed set.

Corollary 2.1. If A, B are (γ, δ)-BSC sets then $(A \cap B)$ is not necessarily a (γ, δ)-BSC set.

Proof. It is clear from Theorem 2.2.

Corollary 2.2. If $A \in BSC(X)$ and $B \in BSC(Y)$ then $(A \times B)$ is $Product$-2-(γ, δ)-BSC-set.

Proof. Proof is straight forward in view of $(\tau\theta)_1$-$(A \times B)^*$=τ_1-$A^* \times \theta_1$-B^*.

Definition 2.2. A is said to be I-τ_2-gs-closed set in $(X, \tau_1, \tau_2, I, \gamma, \delta)$ if τ_1-$A^* \subseteq \tau_2^s$-U whenever $A \subseteq \tau_2^s$-U.

A is said to be I^s-τ_2-gs-closed set in $(X, \tau_1, \tau_2, I, \gamma, \delta)$ if τ_1-$A^* \subseteq (\tau_2^s$-$U) \cap S$ whenever $A \subseteq (\tau_2^s$-$U) \cap S$ where $S \subseteq X$.

Theorem 2.4. Let $(X, \tau_1, \tau_2, I, \gamma, \delta)$ be a bitopological space and $A \subseteq X, F \subseteq X$.If A is I-τ_2-gs closed set and F is τ_2-closed set and τ_1^*-closed set in $(X, \tau_1, \tau_2, I, \gamma, \delta)$ then $(A \cap F)$ is a I-τ_2-gs-closed set.

Proof. Let $(A \cap F) \subseteq \tau_2^s$-$U$. Then $A \subseteq (\tau_2^s$-$U) \cup (X \setminus F)$. Then clearly $(\tau_2^s$-$U) \cup (X \setminus F)$ is τ_2 semi open. Also τ_1-$F^* \subseteq F$.

Now $(\tau_1$-$A^*) \subseteq (\tau_2^s$-$U) \cup (X \setminus F)$ implies $(\tau_1$-$A^*) \cap F \subseteq \tau_2^s$-$U$.

Then τ_1-$(A \cap F)^* \subseteq (\tau_1$-$A^*) \cap (\tau_1$-$F^*) \subseteq (\tau_1$-$A^*) \cap F \subseteq (\tau_2^s$-$U)$.Thus $(A \cap F)$ is a I-τ_2-gs-closed set.

Corollary 2.3. Let $(X, \tau_1, \tau_2, I, \gamma, \delta)$ be a bitopological space and $A \subseteq X, F \subseteq X$. A is I^s-τ_2-gs-closed set, F is τ_2-closed set and τ_1^*-closed set in (X, τ_1, τ_2). If $(U \cap S)$ is τ_2-clopen set for all τ_2-semi-open sets U and $S \subseteq X$ then $(A \cap F)$ is a I^s-τ_2-gs-closed set.

Proof. Let $(A \cap F) \subseteq (\tau_2^s\text{-}U) \cap S$. Then $(\tau_2^s\text{-}U) \cap S$ is τ_2-clopen, so it is also τ_2-semi-open set. Then proceeding as previous we have the result.

Corollary 2.4. If A is $I\text{-}\tau_2\text{-}gs$ closed subset of $(X, \tau_1, \tau_2, I, \gamma, \delta)$ and $F \subseteq X$, F is τ_2-closed and τ_1^*-closed in (X, τ_1, τ_2). If $(U \cap S)$ is τ_2-clopen set for all τ_2-semi-open sets U and $S \subseteq X$ then $(A \cap F)$ is a $I^s\text{-}\tau_2\text{-}gs$-closed set.

We procure the following two results due to Dontchev etal [1] to establish Theorem 2.5.

Lemma 2.1. If A and B are subsets of (X, τ, I), then $(A \cap B)^*(I) \subseteq A^*(I) \cap B^*(I)$.

A subset S of a topological space (X, τ, I) is a topological space with an ideals $I_S = \{F \cap S : F \in I\}$.
Lemma 2.2. Let (X, τ, I) be a topological space and $A \subseteq S \subseteq X$ then $A^*(I_s, \tau|_S) = A^*(I, \tau) \cap S$ holds.

Theorem 2.5. Let $(X, \tau_1, \tau_2, I, \gamma, \delta)$ be a bitopological space and $A \subseteq S \subseteq X$. If A is τ_1^*-closed set and $I_S\text{-}\tau_2|_S\text{-}gs$-closed set in $(S, \tau_1|_S, \tau_2|_S)$ contained in τ_2-open set but not contained in any τ_2-semi-open sets which is not τ_2-open set, then A is $I^s\text{-}\tau_2\text{-}gs$-closed set in $(X, \tau_1, \tau_2, I, \gamma, \delta)$.

Proof. A is $I_S\text{-}\tau_2|_S\text{-}gs$-closed set in $(S, \tau_1|S, \tau_2|_S)$. So, $A^*(I_S, \tau_1|_S) \subseteq \tau_2|_S\text{-}V$ whenever $A \subseteq \tau_2|_S\text{-}V$. This implies $A^*(I, \tau_1) \cap S \subseteq S \cap \tau_2\text{-}U$ Whenever $A \subseteq S \cap \tau_2\text{-}U$ (say). As we know that every τ_i,$i = 1, 2$ open sets are τ_i,$i = 1, 2$ semi-open sets and thus $\tau_1\text{-}A^* \subseteq (\tau_2^s\text{-}U) \cap S$ Whenever $A \subseteq (\tau_2^s\text{-}U) \cap S$. This establishes the result.

Definition 2.3. Let $A \subseteq X$, then $\omega(A)= \cup \{G|\ G \subseteq A, G$ is a $(\gamma, \delta)\text{-}BSC$-set$\}$

Proposition 2.1. Let $A \subseteq X$ then

(i) $\omega(A) \subseteq \tau_i\text{-}cl^*(A)$

(ii) $\omega(A) \subseteq \tau_i\text{-}cl(A)$

Theorem 2.6. If τ_i-$cl^*(A) \subseteq \omega(A)$ then A is τ_i^*-closed.

Proof. τ_i-$cl^*(A) \subseteq \omega(A)$ then $\omega(A) = \cup G$ where $G \subseteq A$ and G is (γ, δ)-BSC-set. then τ_i-$cl^*(A) \subseteq \cup G \subseteq A$ then τ_i-$cl^*(A) \subseteq A$.Hence the result.

Theorem 2.7. If $(X, \tau_1, \tau_2, I, \gamma, \delta)$ be a bitopological space and $A \subseteq X$ and

(i) τ_1^s-$cl^*(A) \subseteq \omega(A)$.

(ii) A is (γ, δ)-BSC-set.

(iii) τ_1^s-$cl^*(A)$ is (γ, δ)-BSC-set.

Then $(i) \Rightarrow (ii) \Rightarrow (iii)$.

Proof. (i)⇒(ii) τ_1^s-$cl^*(A) \subseteq \omega(A)$ implies $A \subseteq \omega(A)$. So, $A \subseteq (\cup G)$ where $G \subseteq A$ and G is (γ, δ)-BSC-set.

Then $A = A \cap (\cup G) = \cup(A \cap G) = \cup G$, which is (γ, δ)-BSC-set. Thus A is (γ, δ)-BSC-set.

(ii)⇒(iii) A is (γ, δ)-BSC-set then clearly by Theorem 2.3, τ_1^s-$cl^*(A)$ is (γ, δ)-BSC-set.

Corollary 2.5. If A is (γ, δ)-BSC-set then the inclusion τ_1^s-$cl^*(A) \subseteq \omega(A)$ may not hold in general.

This result can be verified by the following example.

Example 2.4. Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$, $\tau_2 = \{\emptyset, X, \{b\}\}$, I=$\{\emptyset, \{c\}\}$. We define $U^{\tau_2^s - \gamma} = \tau_2^s$-$U$ and $U^{\tau_2^s - \delta} = \tau_2$-$cl(\tau_2^s$-$U)$. Let $A = \{a, b\}$ then τ_1^s-$cl^*(A) = X$. Clearly A is $(\gamma_{id}, \delta_{cl})$-$BSC$-set. But A is the only $(\gamma_{id}, \delta_{cl})$-$BSC$-set which is contained in A. thus $\omega(A) = A$ and τ_1^s-$cl^*(A)$ is not a subset of $\omega(A)$. Hence the result.

Definition 2.4. Let $(X, \tau_1, \tau_2, I, \gamma, \delta)$ be a bitopological space. Then the Kernel of $(X, \tau_1, \tau_2, I, \gamma, \delta)$ is denoted by Ker(γ, δ) which is defined by Ker(γ, δ)=$\{A \subseteq X \mid \tau_1$-$A^* = \emptyset, A \notin BSC(X)\}$.

Let $(X,\tau_1,\tau_2,I,\gamma,\delta)$ be a bitopological space. Then Image of $(X,\tau_1,\tau_2,I,\gamma,\delta)$ is denoted by Img(γ,δ) which is defined by Img(γ,δ)= $\{A\subseteq X \mid \tau_1\text{-}A^*\neq\emptyset, A\notin BSC(X)\}$.

Remark 2.2. Ker$(\gamma,\delta)\neq\emptyset$.

Theorem 2.8. If $A\subseteq X$ and $A\in$ Ker(γ,δ) then $\tau_j|_A\subseteq$ Ker(γ,δ), $j=1,2$.

Proof. Let $B\in\tau_j|_A$ then $B=(A\cap\tau_j\text{-}U)$, then $\tau_1\text{-}B^*=\emptyset$ and clearly $B\notin BSC(X)$. Thus $B\in$ Ker(γ,δ), $j=1,2$. Hence the proof.

Theorem 2.9. If $A\subseteq X$ and $A\in$ Img(γ,δ) then $\tau_j|_A\setminus\{\emptyset\}\subseteq$ Img(γ,δ), $j=1,2$.

Proof. Let $B\in\tau_j|_A\setminus\{\emptyset\}$, then $B=(A\cap\tau_j\text{-}U)\neq\emptyset$ and $\tau_1\text{-}A^*\neq\emptyset$, then clearly $\tau_1\text{-}B^*\neq\emptyset$ and $B\notin BSC(X)$. This establishes the result.

Definition 2.5. Let $A\subseteq X$, then (γ,δ)-closure of A and (γ,δ)-interior of A is denoted by (γ,δ)-$cl(A)$= $\cap$ $\{C|A\subseteq C, C\in BSC(X)\}$ and (γ,δ)-$int(A)$= $\cup$ $\{P|P\subseteq A, P\in BSO(X)\}$.

If $(X,\tau_1,\tau_2,I_1,\gamma,\delta)$ and $(Y,\theta_1,\theta_2,I_2,\zeta,\eta)$ be two bitopological spaces and $A\subseteq X, B\subseteq Y$ then (γ,δ)-$cl(A\times B)$= $\cap$ $\{C\times D|A\times B\subseteq C\times D; C\in BSC(X), D\in BSC(Y)\}$ and (γ,δ)-$int(A\times B)$= $\cup$ $\{P\times Q|P\times Q\subseteq A\times B; P\in BSO(X), Q\in BSO(Y)\}$.

Proposition 2.2. If $(X,\tau_1,\tau_2,I_1,\gamma,\delta)$ and $(Y,\theta_1,\theta_2,I_2,\zeta,\eta)$ be two bitopological spaces, then

(i) $A\subseteq(\gamma,\delta)$-$cl(A)$ where $A\subseteq X$.

(ii)$A\subseteq B\Rightarrow(\gamma,\delta)$-$cl(A)\subseteq(\gamma,\delta)$-$cl(B)$ where $A,B\subseteq X$.

(iii) (γ,δ)-$cl(A\cup B)$=(γ,δ)-$cl(A)\cup(\gamma,\delta)$-$cl(B)$.

(iv) (γ,δ)-$cl(A\cap B)\subseteq(\gamma,\delta)$-$cl(A)\cap(\gamma,\delta)$-$cl(B)$.

(v) (γ,δ)-$cl(A\times B)$=(γ,δ)-$cl(A)\times$ (ζ,η)-$cl(B)$ where $A\subseteq X, B\subseteq Y$.

Proposition 2.3. If $(X, \tau_1, \tau_2, I_1, \gamma, \delta)$ and $(Y, \theta_1, \theta_2, I_2, \zeta, \eta)$ be two bitopological spaces,then

(i) (γ, δ)-$int(A) \subseteq A$ where $A \subseteq X$.

(ii) $A \subseteq B \Rightarrow (\gamma, \delta)$-$int(A) \subseteq (\gamma, \delta)$-$int(B)$ where $A, B \subseteq X$.

(iii) (γ, δ)-$int(A \cap B)$=(γ, δ)-$int(A) \cap (\gamma, \delta)$-$int(B)$.

(iv) (γ, δ)-$int(A \cup B) \supseteq (\gamma, \delta)$-$int(A) \cup (\gamma, \delta)$-$int(B)$.

(v)) (γ, δ)-$int(A \times B)$=(γ, δ)-$int(A)\times$ (ζ, η)-$int(B)$ where $A \subseteq X, B \subseteq Y$.

Theorem 2.10. Let $(X, \tau_1, \tau_2, I, \gamma, \delta)$ be a bitopological space, $A \subseteq X$ and $x \in X$. Then $x \in (\gamma, \delta)$-$cl(A)$ if and only if $U \cap A \neq \emptyset$ where $x \in U, U \in BSO(X)$.

Proof. Let $x \in (\gamma, \delta)$-$cl(A)$. If possible, let $x \in U, U \in BSO(X)$ such that $U \cap A$=$\emptyset$. So, $A \subseteq (X \setminus U)$, then clearly $x \notin U$, which is a contradiction. So, $U \cap A \neq \emptyset$ where $x \in U, U \in BSO(X)$.

Conversely, let $U \cap A \neq \emptyset$ where $x \in U, U \in BSO(X)$. If possible let $x \notin (\gamma, \delta)$-$cl(A)$. Then $x \notin \cap C$, where $A \subset C$, C is a (γ, δ)-BSC set. so $x \notin A$ implies $U \cap A$=$\emptyset$ - a contradiction. Hence $x \in (\gamma, \delta)$-$cl(A)$.

Corollary 2.6. (i) (γ, δ)-$cl(X \setminus A)$= $X \setminus (\gamma, \delta)$-$int(A)$.

(ii)(γ, δ)-$int(X \setminus A)$=$X \setminus (\gamma, \delta)$-$cl(A)$.

Definition 2.6. A is called (γ, δ)-closed if (γ, δ)-$cl(A) = A$ and then $(X \setminus A)$ is called (γ, δ)-open.

A is called *Product*-2-(γ, δ)-closed if $(\gamma, \delta) - cl(A \times B) = (A \times B)$ and then $(X \times Y \setminus (A \times B))$ is called *Product*-2-(γ, δ)-open.

A is called (γ, δ)-clopen set if A is both (γ, δ)-closed and (γ, δ)-open.

Theorem 2.11. Let $(X, \tau_1, \tau_2, I, \gamma, \delta)$ be a bitopological space and $A \subseteq X$.

A is (γ,δ)-open if and only if $A = (\gamma,\delta)$-$int(A)$.

Theorem 2.12. Let $(X,\tau_1,\tau_2,I,\gamma,\delta)$ be a bitopological space. If A is (γ,δ)-BSC set; then (γ,δ)-$cl(A)\setminus A$ does not contain any non-empty (γ,δ)-open set.

Proof. Let $F \neq \emptyset$, F is (γ,δ)-open set. If possible, let us assume that $F \subseteq (\gamma,\delta)$-$cl(A) \setminus A$ Then $F \subseteq (\gamma,\delta)$-$cl(A)$ but $A \subseteq (X \setminus F)$. Then, (γ,δ)-$cl(A) \subseteq (X \setminus F)$. Thus $F \subseteq (\gamma,\delta)$-$cl(A) \cap (X \setminus (\gamma,\delta)$-$cl(A)))$=$\emptyset$. So F=$\emptyset$, -a contradiction. Hence the result.

Definition 2.7. Let $(X,\tau_1,\tau_2,I,\gamma,\delta)$ be a bitopological space and $A \subseteq X$. Then we define $Ker(A)$= $\cap\{U \in BSO(X) : A \subseteq U\}$.

If $A \in BSO(X)$, then it is easy to verify that $Ker(A) = A$.

Theorem 2.13. Let $(X,\tau_1,\tau_2,I,\gamma,\delta)$ be a bitopological space and $A \subseteq X$. Then following results hold-

(i) $A \subseteq Ker(A)$.

(ii) $A \subseteq B \Rightarrow Ker(A) \subseteq Ker(B)$.

(iii) $x \in Ker(A)$ if and only if $A \cap M \neq \emptyset$ where $x \in M$ and $M \in BSC(X)$.

Proof. proofs of (i) and (ii) are easy and so omitted.

(3) Let $x \in Ker(A)$. If possible let $A \cap M$= $\emptyset$ where $x \in M$ and $M \in BSC(X)$. Then $A \subseteq (X \setminus M)$. So, $Ker(A) \subseteq Ker(X \setminus M)$=$(X \setminus M)$. Which implies $x \notin M$, a contradiction. Hence $A \cap M \neq \emptyset$.

Conversely, let $x \notin Ker(A)$ and let $A \cap M \neq \emptyset$ where $x \in M$ and $M \in BSC(X)$. Then $\exists\, U \in BSO(X), x \notin U, A \subseteq U$. So, $(X \setminus U) \in BSC(X)$. Thus $x \notin A$. Now by the assumption, $A \cap (X \setminus U) \neq \emptyset$ i.e. A is not subset of U, a contradiction. Thus $x \in Ker(A)$.

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