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On (i, j)-ω-preopen sets

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Abstract

In this paper, we introduce and study the notion of (i, j)-ω-preopen sets as a generalization of (i, j)-preopen sets in bitopological space.

Keywords : *Bitopological spaces, (i, j)-preopen sets, (i, j)-precontinuous functions.*

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1. Introduction and Preliminaries

Generalized open sets play a very important role in General Topology and they are now the research topics of many topologists worldwide. Indeed a significant theme in General Topology and Real analysis concerns the variously modified forms of continuity, separation axioms etc. by utilizing generalized open sets. The concept of a bitopological space was introduced by Kelly [4]. On the other hand Jelic [2] introduced the concept of preopen sets in bitopological spaces. In this paper, we introduce and study the notion of (i, j)-ω-preopen sets as a generalization of (i, j)-preopen sets in bitopological spaces. Throughout this paper, spaces means bitopological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of X, the closure and the interior of A are denoted by (A) and (A), respectively. A subset A of a bitopological space (X, τ_1, τ_2) is said to be (i, j)-preopen [2] if $A \subset \tau_i$-$(\tau_j$-$(A))$, where $i, j = 1, 2$ and $i \neq j$. The complement of an (i, j)-preopen set is said to be (i, j)-preclosed set ([3], [5]). The (i, j)-preclosure [5] of A, denoted by (i, j)-$p(A)$, is defined by the intersection of all (i, j)-preclosed sets containing A. The (i, j)-preinterior [6] of A, denoted by (i, j)-$p(A)$, is defined by the union of all (i, j)-preopen sets contained in A. A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be (i, j)-precontinuous ([3], [5]) if the inverse image of every σ_i-open set in (Y, σ_1, σ_2) is (i, j)-preopen in (X, τ_1, τ_2), where $i \neq j$, $i, j = 1, 2$.

2. (i, j)-ω-preopen sets

Definition 2.1. *A subset A is said to be (i, j)-ω-preopen if for each $x \in A$ there exists an (i, j)-preopen set U_x containing x such that $U_x \backslash A$ is a countable set. The complement of an (i, j)-ω-preopen subset is said to be (i, j)-ω-preclosed.*
The family of all (i, j)-ω-preopen (resp. (i, j)-ω-preclosed) subsets of a space (X, τ_1, τ_2) is denoted by (i, j)-$\omega PO(X)$ (resp. (i, j)-$\omega PC(X)$). Also, The family of all (i, j)-ω-preopen sets of (X, τ_1, τ_2) containing x is denoted by (i, j)-$\omega PO(X, x)$.

It is clear that every (i, j)-preopen set is (i, j)-ω-preopen. The following example shows that the converse is not true in general.

Example 2.2. *Let $X = \{a, b, c\}$, $\tau_1 = \{, \{a\}, X\}$ and $\tau_2 = \{, \{b\}, X\}$. Then $\{a, c\}$ is (i, j)-ω-preopen but not (i, j)-preopen in (X, τ_1, τ_2).*

Lemma 2.3. *A subset A of a bitopological space (X, τ_1, τ_2) is (i,j)-ω-preopen if and only if for every $x \in A$, there exists an (i,j)-preopen subset U_x containing x and a countable subset C such that $U_x \backslash C \subset A$.*

Proof. Let A be (i,j)-ω-preopen and $x \in A$, then there exists an (i,j)-ω-preopen subset U_x containing x such that $U_x \backslash A$ is countable. Let $C = U_x \backslash A = U_x \cap (X \backslash A)$. Then $U_x \backslash C \subset A$. Conversely, let $x \in A$. Then there exists an (i,j)-ω-preopen subset U_x containing x and a countable subset C such that $U_x \backslash C \subset A$. Thus, $U_x \backslash A \subset C$ and $U_x \backslash A$ is countable. $\square$

Theorem 2.4. *Let (X, τ_1, τ_2) be a bitopological space and $C \subset X$. If C is (i,j)-ω-preclosed, then $C \subset K \cup B$ for some (i,j)-ω-preclosed subset K and a countable subset B.*

Proof. If C is (i,j)-ω-preclosed, then $X \backslash C$ is (i,j)-ω-preopen and hence for every $x \in X \backslash C$, there exists an (i,j)-ω-preopen set U containing x and a countable set B such that $U \backslash B \subset X \backslash C$. Thus $C \subset X \backslash (U \backslash B) = X \backslash (U \cap (X \backslash B)) = (X \backslash U) \cup B$. Let $K = X \backslash U$. Then K is (i,j)-ω-preclosed such that $C \subset K \cup B$. $\square$

Proposition 2.5. *The union of any family of (i,j)-ω-preopen sets is (i,j)-ω-preopen.*

Proof. If $\{A_\alpha : \alpha \in \Lambda\}$ is a collection of (i,j)-ω-preopen subsets of X, then for every $x \in \bigcup_{\alpha \in \Lambda} A_\alpha$, $x \in A_\gamma$ for some $\gamma \in \Lambda$. Hence there exists an (i,j)-preopen subset U of X containing x such that $U \backslash A_\gamma$ is countable. Now as $U \backslash \bigcup_{\alpha \in \Lambda} A_\alpha \subset U \backslash A_\gamma$ and thus $U \backslash \bigcup_{\alpha \in \Lambda} A_\alpha$ is countable. Therefore, $\bigcup_{\alpha \in \Lambda} A_\alpha$ is (i,j)-ω-preopen. $\square$

Definition 2.6. *The union of all (i,j)-ω-preopen sets contained in $A \subset X$ is called the (i,j)-ω-preinterior of A, and is denoted by (i,j)-$\omega p(A)$. The intersection of all (i,j)-ω-preclosed sets of X containing A is called the (i,j)-ω-preclosure of A, and is denoted by (i,j)-$\omega p(A)$.*

Theorem 2.7. *Let A and B be subsets of (X, τ_1, τ_2). Then the following properties hold:*

(i) (i,j)-$\omega p(A)$ is the largest (i,j)-ω-preopen subset of X contained in A.

(ii) A *is* (i,j)*-*ω*-preopen if and only if* $A = (i,j)$*-*$\omega p(A)$*.*

(iii) (i,j)-$\omega p((i,j)$-$\omega p(A)) = (i,j)$-$\omega p(A)$*.*

(iv) If $A \subset B$*, then* (i,j)-$\omega p(A) \subset (i,j)$-$\omega p(B)$*.*

(v) (i,j)-$\omega p(A \cap B) \subset (i,j)$-$\omega p(A) \cap \subset (i,j)$-$\omega p(B)$*.*

(vi) (i,j)-$\omega p(A) \cup (i,j)$-$\omega p(B) \subset (i,j)$-$\omega p(A \cup B)$*.*

Proof. (v). Since $A \cap B \subset A$ and $A \cap B \subset B$, by (iv), we have (i,j)-$\omega p(A \cap B) \subset (i,j)$-$\omega p(A)$ and (i,j)-$\omega p(A \cap B) \subset (i,j)$-$\omega p(B)$. Therefore, (i,j)-$\omega p(A \cap B) \subset (i,j)$-$\omega p(A) \cap (i,j)$-$\omega p(B)$.
(vi). We have (i,j)-$\omega p(A) \subset (i,j)$-$\omega p(A \cup B)$ and (i,j)-$\omega p(B) \subset (i,j)$-$\omega p(A \cup B)$. Then we obtain (i,j)-$\omega p(A) \cup (i,j)$-$\omega p(B) \subset (i,j)$-$\omega p(A \cup B)$. The other proof are obvious. □

Theorem 2.8. *Let* A *and* B *be subsets of* (X, τ_1, τ_2)*. Then the following properties hold:*

(i) (i,j)-$\omega p(A)$ *is the smallest* (i,j)*-*ω*-preclosed subset of* X *containing* A*.*

(ii) A *is* (i,j)*-*ω*-preclosed if and only if* $A = (i,j)$-$\omega p(A)$*.*

(iii) (i,j)-$\omega p((i,j)$-$\omega p(A)) = (i,j)$-$\omega p(A)$*.*

(iv) If $A \subset B$*, then* (i,j)-$\omega p(A) \subset (i,j)$-$\omega p(B)$*.*

(v) (i,j)-$\omega p(A) \cup (i,j)$-$\omega p(B) \subset (i,j)$-$\omega p(A \cup B)$*.*

(vi) (i,j)-$\omega p(A \cap B) \subset (i,j)$-$\omega p(A) \cap (i,j)$-$\omega p(B)$*.*

Proof. The proofs follows from the definitions. □

Theorem 2.9. *Let* (X, τ_1, τ_2) *be a bitopological space and* $A \subset X$*. A point* $x \in (i,j)$-$\omega p(A)$ *if and only if* $U \cap A \neq \emptyset$ *for every* $U \in (i,j)$-$\omega PO(X, x)$*.*

Proof. Suppose that $x \in (i,j)$-$\omega p(A)$. We shall show that $U \cap A \neq \emptyset$ for every $U \in (i,j)$-$\omega PO(X,x)$. Suppose that there exists $U \in (i,j)$-$\omega PO(X,x)$ such that $U \cap A = \emptyset$. Then $A \subset X\backslash U$ and $X\backslash U$ is (i,j)-ω-preclosed. Since $A \subset X\backslash U$, (i,j)-$\omega p(A) \subset (i,j)$-$\omega p(X\backslash U)$. Since $x \in (i,j)$-$\omega p(A)$, we have $x \in (i,j)$-$\omega p(X\backslash U)$. Since $X\backslash U$ is (i,j)-ω-preclosed, we have $x \in X\backslash U$; hence $x \notin U$, which contradicts the fact that $x \in U$. Therefore, $U \cap A \neq \emptyset$. Conversely, suppose that $U \cap A \neq \emptyset$ for every $U \in (i,j)$-$\omega PO(X,x)$. We shall show that $x \in (i,j)$-$\omega p(A)$. Suppose that $x \notin (i,j)$-$\omega p(A)$. Let $U = X\backslash(i,j)$-$\omega p(A)$, then $U \in (i,j)$-$\omega PO(X,x)$ such that $U \cap A = (X\backslash(i,j)$-$\omega p(A)) \cap A \subset (X\backslash A) \cap A = \emptyset$. This is a contradicition to $U \cap A \neq \emptyset$; hence $x \in (i,j)$-$\omega p(A)$. $\square$

Theorem 2.10. *Let (X, τ_1, τ_2) be a bitopological space and $A \subset X$. Then the following propeties hold:*

(i) (i,j)-$\omega p(X\backslash A) = X\backslash(i,j)$-$\omega p(A)$;

(i) (i,j)-$\omega p(X\backslash A) = X\backslash(i,j)$-$\omega p(A)$.

Proof. (i). Let $x \in X\backslash(i,j)$-$\omega p(A)$. Since $x \notin (i,j)$-$\omega p(A)$, there exists $V \in (i,j)$-$\omega PO(X,x)$ such that $V \cap A = \emptyset$; hence we obtain $x \in (i,j)$-$\omega p(X\backslash A)$. This shows that $X\backslash(i,j)$-$\omega p(A) \subset (i,j)$-$\omega p(X\backslash A)$. Let $x \in (i,j)$-$\omega p(X\backslash A)$. Since (i,j)-$\omega p(X\backslash A) \cap A = \emptyset$, we obtain $x \notin (i,j)$-$\omega p(A)$; hence $x \in X\backslash(i,j)$-$\omega p(A)$. Therefore, we obtain (i,j)-$\omega p(X\backslash A) = X\backslash(i,j)$-$\omega p(A)$.
(ii). Follows from (i). $\square$

Definition 2.11. *A subset B_x of a bitopological space (X, τ_1, τ_2) is said to be an (i,j)-ω-preneighbourhood of a point $x \in X$ if there exists an (i,j)-ω-preopen set U such that $x \in U \subset B_x$.*

Theorem 2.12. *A subset of a bitopological space (X, τ_1, τ_2) is (i,j)-ω-preopen if and only if it is an (i,j)-ω-preneighbourhood of each of its points.*

Proof. Let G be an (i,j)-ω-preopen set of X. Then by definition, it is clear that G is an (i,j)-ω-preneighbourhood of each of its points, since for every $x \in G$, $x \in G \subset G$ and G is (i,j)-ω-preopen. Conversely, suppose G is an (i,j)-ω-preneighbourhood of each of its points. Then for each $x \in G$, there exists $S_x \in (i,j)$-$\omega PO(X)$ such that $S_x \subset G$. Then $G = \bigcup\{S_x : x \in G\}$. Since each S_x is (i,j)-ω-preopen, G is (i,j)-ω-preopen in (X, τ_1, τ_2). $\square$

Theorem 2.13. *If each nonempty (i,j)-preopen set of a bitopological space (X,τ_1,τ_2) is uncountable, then (i,j)-$p(A) = (i,j)$-$\omega p(A)$ for each subset $A \in \tau_1 \cap \tau_2$.*

Proof. Clearly (i,j)-$\omega p(A) \subset (i,j)$-$p(A)$. On the other hand, let $x \in (i,j)$-$p(A)$ and B be an (i,j)-ω-preopen subset containing x. Then by Lemma 2.3, there exists an (i,j)-preopen set V containing x and a countable set C such that $V \backslash C \subset B$. Thus $(V \backslash C) \cap A \subset B \cap A$ and so $(V \cap A) \backslash C \subset B \cap A$. Since $x \in V$ and $x \in (i,j)$-$p(A)$, $V \cap A \neq \emptyset$ and $V \cap A$ is (i,j)-preopen since V is (i,j)-preopen and $A \in \tau_1 \cap \tau_2$. By the hypothesis each nonempty (i,j)-preopen set of X is uncountable and so is $(V \cap A) \backslash C$. Thus $B \cap A$ is uncountable. Therefore, $B \cap A \neq \emptyset$ which means that $x \in (i,j)$-$\omega p(A)$. □

Corollary 2.14. *If each nonempty (i,j)-preclosed set of a bitopological space (X,τ_1,τ_2) is uncountable, then (i,j)-$p(A) = (i,j)$-$\omega p(A)$ for each $A \in \tau_1 \cap \tau_2$.*

Theorem 2.15. *If every (i,j)-preopen subset of X is τ_i-open in (X,τ_1,τ_2), then $(X,(i,j)$-$\omega PO(X))$ is a topological space.*

Proof. (i). We have $\emptyset$, $X \in (i,j)$-$\omega PO(X)$. (ii). Let $U, V \in (i,j)$-$\omega PO(X)$ and $x \in U \cap V$. Then there exist (i,j)-preopen sets $G, H \in X$ containing x such that $G \backslash U$ and $H \backslash V$ are countable. And $(G \cap H) \backslash (U \cap V) = (G \cap H) \cap ((X \backslash U) \cup (X \backslash V)) \subset (G \cap (X \backslash U)) \cup (H \cap (X \backslash V))$. Hence $(G \cap H) \backslash (U \cap V)$ is countable and by hypothesis, the intersection of two (i,j)-preopen sets is (i,j)-preopen. Hence $U \cap V \in (i,j)$-$\omega PO(X)$. (iii). Let $\{U_i : i \in I\}$ be any family of (i,j)-ω-preopen sets of X. Then, by Proposition 2.5 $\bigcup_{i=1}^{n} U_i$ is (i,j)-ω-preopen. □

3. (i,j)-ω-precontinuous functions

Definition 3.1. *A function $f : (X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ is said to be (i,j)-ω-precontinuous if the inverse image of every σ_i-open set of Y is (i,j)-ω-preopen in X, where $i \neq j$, i,j=1, 2.*

It is clear that every (i,j)-precontinuous function is (i,j)-ω-precontinuous but not conversely.

Example 3.2. *Let $X = \{a,b,c\}$, $\tau = \{,\{a\},X\}$ and $\sigma = \{,\{a,c\},X\}$. Clearly the identity function $f : (X,\tau) \to (X,\sigma)$ is (i,j)-ω-precontinuous but not (i,j)-precontinuous.*

Theorem 3.3. *For a function $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$, the following statements are equivalent:*

(i) f is (i,j)-ω-precontinuous;

(ii) For each point x in X and each σ_i-open set F in Y such that $f(x)\in F$, there is an (i,j)-ω-preopen set A in X such that $x\in A$, $f(A)\subset F$;

(iii) The inverse image of each σ_i-closed set in Y is (i,j)-ω-preclosed in X;

(iv) For each subset A of X, $f((i,j)\text{-}\omega p(A))\subset\sigma_i\text{-}(f(A))$;

(v) For each subset B of Y, $(i,j)\text{-}\omega p(f^{-1}(B))\subset f^{-1}(\sigma_i\text{-}(B))$;

(vi) For each subset C of Y, $f^{-1}(\sigma_i\text{-}(C))\subset(i,j)\text{-}\omega p(f^{-1}(C))$.

Proof. (i)⇒(ii): Let $x\in X$ and F be a σ_i-open set of Y containing $f(x)$. By (i), $f^{-1}(F)$ is (i,j)-ω-preopen in X. Let $A=f^{-1}(F)$. Then $x\in A$ and $f(A)\subset F$.
(ii)⇒(i): Let F be σ_i-open in Y and let $x\in f^{-1}(F)$. Then $f(x)\in F$. By (ii), there is an (i,j)-ω-preopen set U_x in X such that $x\in U_x$ and $f(U_x)\subset F$. Then $x\in U_x\subset f^{-1}(F)$. Hence $f^{-1}(F)$ is (i,j)-ω-preopen in X.
(i)⇔(iii): This follows due to the fact that for any subset B of Y, $f^{-1}(Y\backslash B)$ $=X\backslash f^{-1}(B)$.
(iii)⇒(iv): Let A be a subset of X. Since $A\subset f^{-1}(f(A))$ we have $A\subset f^{-1}(\sigma_i\text{-}(f(A)))$. Now, $\sigma_i\text{-}(f(A))$ is σ_i-closed in Y and hence $(i,j)\text{-}\omega p(A)\subset f^{-1}(\sigma_i\text{-}(f(A)))$, for $(i,j)\text{-}\omega p(A)$ is the smallest (i,j)-ω-preclosed set containing A. Then $f((i,j)\text{-}\omega p(A))\subset\sigma_i\text{-}(f(A))$.
(iv)⇒(iii): Let F be any σ_i-closed subset of Y. Then $f((i,j)\text{-}\omega p(f^{-1}(F)))$ $\subset\sigma_i\text{-}(f(f^{-1}(F)))\subset\sigma_i\text{-}(F)=F$. Therefore, $(i,j)\text{-}\omega p(f^{-1}(F))\subset f^{-1}(F)$. Consequently, $f^{-1}(F)$ is (i,j)-ω-preclosed in X.
(iv)⇒(v): Let B be any subset of Y. Now, $f((i,j)\text{-}\omega p(f^{-1}(B)))\subset\sigma_i\text{-}(f(f^{-1}(B)))\subset\sigma_i\text{-}(B)$. Consequently, $(i,j)\text{-}\omega p(f^{-1}(B))\subset f^{-1}(\sigma_i\text{-}(B))$.
(v)⇒(iv): Let $B=f(A)$ where A is a subset of X. Then, $(i,j)\text{-}\omega p(A)$ $\subset(i,j)\text{-}\omega p(f^{-1}(B))\subset f^{-1}(\sigma_i\text{-}(B))=f^{-1}(\sigma_i\text{-}(f(A)))$. This shows that $f((i,j)\text{-}\omega p(A))\subset\sigma_i\text{-}(f(A))$.
(i)⇒(vi): Let C be any subset of Y. Clearly, $f^{-1}(\sigma_i\text{-}(C)$ is (i,j)-ω-preopen and we have $f^{-1}(\sigma_i\text{-}(C))\subset(i,j)\text{-}\omega p(f^{-1}\sigma_i\text{-}(C))\subset(i,j)\text{-}\omega p(f^{-1}(C))$.
(vi)⇒(i): Let B be a σ_i-open set in Y. Then $\sigma_i\text{-}(B)=B$ and $f^{-1}(B)\subset$

$f^{-1}(\sigma_i\text{-}(B)) \subset (i,j)\text{-}\omega p(f^{-1}(B))$. Hence we have $f^{-1}(B) = (i,j)\text{-}\omega p(f^{-1}(B))$. This shows that $f^{-1}(B)$ is (i,j)-ω-preopen in X. □

Definition 3.4. *A collection $\{U_\alpha : \alpha \in \Delta\}$ of (i,j)-preopen sets in a bitopological space (X, τ_1, τ_2) is called an (i,j)-preopen cover of a subset B of X if $B \subset \cup\{U_\alpha : \alpha \in \Delta\}$ holds.*

Definition 3.5. *A bitopological space (X, τ_1, τ_2) is said to be (i,j)-preLindelöf if every (i,j)-preopen cover of X has a countable subcover.*
A subset A of a bitopological space X is said to be (i,j)-preLindelöf relative to X if every cover of A by (i,j)-preopen sets of X has a countable subcover.

Theorem 3.6. *If X is a bitopological space such that every (i,j)-preopen subset is (i,j)-preLindelöf relative to X, then every subset is (i,j)-preLindelöf relative to X.*

Proof. Let B be an arbitrary subset of X and let $\{U_\alpha : \alpha \in \Delta\}$ be (i,j)-preopen cover of B. Then the family $\{U_\alpha : \alpha \in \Delta\}$ is an (i,j)-preopen cover of the (i,j)-preopen set $\cup\{U_\alpha : \alpha \in \Delta\}$. Hence by hypothesis there is a countable subfamily $\{U_{\alpha_i} : i \in N\}$ which covers $\cup\{U_\alpha : \alpha \in \Delta\}$. This subfamily is also a cover of the set B. □

Theorem 3.7. *For any bitopological space (X, τ_1, τ_2), the following properties are equivalent:*

(i) X is (i,j)-preLindelöf.

(ii) Every countable cover of X by (i,j)-ω-preopen sets has a countable subcover.

Proof. $(i) \Rightarrow (ii)$: Let $\{U_\alpha : \alpha \in \Delta\}$ be any cover of X by (i,j)-ω-preopen sets of X. For each $x \in X$, there exists $\alpha(x) \in \Delta$ such that $x \in U_{\alpha(x)}$. Since $U_\alpha(x)$ is (i,j)-ω-preopen,there exists an (i,j)-preopen set $V_{\alpha(x)}$ such that $x \in V_{\alpha(x)}$ and $V_{\alpha(x)} \backslash U_{\alpha(x)}$ is countable. The family $\{V_{\alpha(x)} : x \in X\}$ is an (i,j)-preopen cover of X and X is (i,j)-preLindelöf. There exists a countable subset, say $\alpha(x_1), \alpha(x_2), ...\alpha(x_n)$,....... such that $X = \cup\{V_{\alpha(x_i)} : i \in N\}$. Now, we have $X = \bigcup_{i \in N} \{V_{\alpha(x_i)} \backslash U_{\alpha(x_i)}) \cup U_{\alpha(x_i)}\}$ $= (\bigcup_{i \in N} (V_{\alpha(x_i)} \backslash U_{\alpha(x_i)}) \cup (\bigcup_{i \in N} U_{\alpha(x_i)}))$. For each $\alpha(x_i)$, $V_{\alpha(x_i)} \backslash U_{\alpha(x_i)}$ is a countable set and there exists a countable subset $\Delta_{\alpha(x_i)}$ of Δ such that $V_{\alpha(x_i)} \backslash U_{\alpha(x_i)} \subset \cup\{U_\alpha : \alpha \in \Delta_{\alpha(x_i)}\}$. Therefore, we have $X \subset \bigcup_{i\in N}(\cup\{U_\alpha : \alpha \in \Delta_{\alpha(x_i)}\}) \cup (\bigcup_{i\in N} U_{\alpha(x_i)})$.

$(ii) \Rightarrow (i)$: Since every (i,j)-preopen is (i,j)-ω-preopen, the proof is obvious. □

Definition 3.8. *A bitopological space (X, τ_1, τ_2) is called pairwise Lindelöf [1] if each pairwise open cover of X has a countable subcover.*

Theorem 3.9. *Let f be an (i,j)-ω-precontinuous function from a space (X, τ_1, τ_2) onto a space (Y, σ_1, σ_2). If X is (i,j)-preLindelöf, then Y is pairwise Lindelöf.*

Proof. Let $\{V_\alpha : \alpha \in \Lambda\}$ be a countable cover of Y by σ_i-open sets. Then $\{f^{-1}(V_\alpha) : \alpha \in \Lambda\}$ is an (i,j)-ω-preopen cover of X. Since X is (i,j)-preLindelöf, there exists a countable subset Λ_0 of Λ such that $X = \cup\{f^{-1}(V_\alpha) : \alpha \in \Lambda_0\}$; hence $Y = \cup\{V_\alpha : \alpha \in \Lambda_0\}$. Therefore Y is pairwise preLindelöf. □

Definition 3.10. *A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be:*

(i) *(i,j)-ω-preopen if $f(U)$ is a (i,j)-ω-preopen set of Y for every τ_i-open set U of X.*

(ii) *(i,j)-ω-preclosed if $f(U)$ is a (i,j)-ω-preclosed set of Y for every τ_i-closed set U of X.*

Theorem 3.11. *For a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following statements are equivalent:*

(i) *f is (i,j)-ω-preopen;*

(ii) *$f(\tau_i\text{-}(U)) \subset (i,j)\text{-}\omega p(f(U))$ for each subset U of X;*

(iii) *$\tau_i\text{-}(f^{-1}(V)) \subset f^{-1}((i,j)\text{-}\omega p(V))$ for each subset V of Y.*

Proof. $(i) \Rightarrow (ii)$: Let U be any subset of X. Then τ_i-(U) is a τ_i-open set of X. Then $f(\tau_i$-$(U))$ is a (i,j)-ω-preopen set of Y. Since $f(\tau_i$-$(U)) \subset f(U)$, $f(\tau_i$-$(U)) = (i,j)$-$\omega p(f(\tau_i$-$(U))) \subset (i,j)$-$\omega p(f(U))$.
$(ii) \Rightarrow (iii)$: Let V be any subset of Y. Then $f^{-1}(V)$ is a subset of X. Hence $f(\tau_i$-$(f^{-1}(V))) \subset (i,j)$-$\omega p(f(f^{-1}(V))) \subset (i,j)$-$\omega p(V))$. Then τ_i-$(f^{-1}(V)) \subset f^{-1}(f(\tau_i$-$(f^{-1}(V)))) \subset f^{-1}((i,j)$-$\omega p(V))$.
$(iii) \Rightarrow (i)$: Let U be any τ_i-open set of X. Then τ_i-$(U) = U$ and $f(U)$ is a subset of Y. Now, $V = \tau_i$-$(V) \subset \tau_i$-$(f^{-1}(f(V))) \subset f^{-1}((i,j)$-$\omega p(f(V)))$. Then $f(V) \subset f(f^{-1}((i,j)$-$\omega p(f(V)))) \subset (i,j)$-$\omega p(f(V))$ and (i,j)-$\omega p(f(V)) \subset f(V)$. Hence $f(V)$ is an (i,j)-ω-preopen set of Y; hence f is (i,j)-ω-preopen. □

Theorem 3.12. *Let $f : (X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ be a function. Then f is an (i,j)-ω-preclosed function if and only if for each subset V of X, (i,j)-$\omega p(f(V)) \subset f(\tau_i$-$(V))$.*

Proof. Let f be an (i,j)-ω-preclosed function and V any subset of X. Then $f(V) \subset f(\tau_i$-$(V))$ and $f(\tau_i$-$(V))$ is an (i,j)-ω-preclosed set of Y. We have (i,j)-$\omega p(f(V)) \subset (i,j)$-$\omega p(f(\tau_i$-$(V))) = f(\tau_i$-$(V))$. Conversely, let V be a τ_i-closed set of X. Then $f(V) \subset (i,j)$-$\omega p(f(V)) \subset f(\tau_i$-$(V)) = f(V)$; hence $f(V)$ is an (i,j)-ω-preclosed subset of Y. Therefore, f is an (i,j)-ω-preclosed function. □

Theorem 3.13. *Let $f : (X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ be a bijection. Then f is an (i,j)-ω-preclosed function if and only if for each subset V of Y, $f^{-1}((i,j)$-$\omega p(V)) \subset \tau_i$-$(f^{-1}(V))$.*

Proof. Let V be any subset of Y. Then by Theorem 3.12, (i,j)-$\omega p(V) \subset f(\tau_i$-$(f^{-1}(V)))$. Since f is bijection, $f^{-1}((i,j)$-$\omega p(V)) = f^{-1}((i,j)$-$\omega p(f(f^{-1}(V)))) \subset f^{-1}(f(\tau_i$-$(f^{-1}(V)))) = \tau_i$-$(f^{-1}(V))$. Conversely, let U be any subset of X. Since f is bijection, (i,j)-$\omega p(f(U)) = f(f^{-1}((i,j)$-$\omega p(f(U))) \subset f(\tau_i$-$(f^{-1}(f(U)))) = f(\tau_i$-$(U))$. Therefore, by Theorem 3.12, f is an (i,j)-ω-preclosed function. □

Theorem 3.14. *Let $f : (X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ be an (i,j)-ω-preopen function. If V is a subset of Y and U is a τ_i-closed subset of X containing $f^{-1}(V)$, then there exists an (i,j)-ω-preclosed set F of Y containing V such that $f^{-1}(F) \subset U$.*

Proof. Let V be any subset of Y and U a τ_i-closed subset of X containing $f^{-1}(V)$, and let $F = Y\backslash(f(X\backslash U))$. Then $f(X\backslash U) \subset f(f^{-1}(Y\backslash V)) \subset Y\backslash V$, then $V \subset F$ and $X\backslash U$ is a τ_i-open set of X. Since f is (i,j)-ω-preopen, $f(X\backslash U)$ is an (i,j)-ω-preopen set of Y. Hence F is an (i,j)-ω-preclosed set of Y and $f^{-1}(F) = f^{-1}(Y\backslash(f(X\backslash U)) \subset U$. □

Theorem 3.15. *Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be an (i,j)-ω-preclosed function. If V is a subset of Y and U is a τ_i-open subset of X containing $f^{-1}(V)$, then there exists (i,j)-ω-preopen set F of Y containing V such that $f^{-1}(F) \subset U$.*

Proof. The proof is similar to that of Theorem 3.14. □

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