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A NEW DEFINITION OF S^* CLOSEDNESS IN L -TOPOLOGICAL SPACES

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Abstract

In this paper, a new notion of S^ closedness in L -topological Spaces is introduced by means of semi-open L -sets and their inequality where L is a complete DeMorgan algebra. This new definition doesn't rely on the structure of basic lattice L . It can be characterized by means of semi-open L -sets and their inequality. When L is completely distributive DeMorgan algebra, its many characterizations are presented.*

Keywords and Phrases : *semiopen L - set, S^* closedness, L -topological space.*

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1. Introduction

Compactness is one of the most important notions in topology. In [1], Azad introduced the concepts of semiopen sets and semiopen continuous functions in L -topology. In [5], Kudri generalized the concept of semi-compactness and S^* closedness in L -topological spaces. However, Kudri's S^* closedness relies on the structure of L which is a completely distributive lattice.

In [10], Shi introduced a new definition of fuzzy compactness in L -topological spaces by means of open L -sets and their inequality where L is a completely distributive *DeMorgan* algebra. This new definition doesn't rely on the structure of basic lattice L .

In this paper, along the line of [10] we shall introduce a new definition of S^* closedness in L -topological spaces by means of semiopen L -sets and their inequality. Also we give characterizations and properties of S^* closedness.

2. Preliminaries

Throughout this paper, $(L, \vee, \wedge, ')$ is a complete *DeMorgan* algebra, X is a nonempty set. L^X is the set of all L -fuzzy sets (L -sets for short) on X . The smallest element and the largest element in L^X are denoted by $\underline{0}$ and $\underline{1}$.

An element a in L is called prime element if $a \geq b \wedge c$ implies $a \geq b$ or $a \geq c$. a in L is called co-prime element if a' is a prime element. The set of non-unit prime elements in L is denoted by $P(L)$. The set of non-zero co-prime elements in L is denoted by $M(L)$.

The binary relation \prec in L is defined as follows: for $a, b \in L$, $a \prec b$ if and only if for every subset $D \subset L$, the relation $b \leq \sup D$ always implies the existence of $d \in D$ with $a \leq d$. In a complete distributive *DeMorgan* algebra L , each element b is a sup of $\{a \in L | a \prec b\}$, $\{a \in L | a \prec b\}$ is called the greatest minimal family of b in the sense of [16], in symbol $\beta(b)$. Moreover for $b \in L$, define $\beta^*(b) = \beta(b) \cap M(L)$, $\alpha(b) = \{a \in L | a' \prec b'\}$ and $\alpha^*(b) = \alpha(b) \cap P(L)$.

For $a \in L$ and $A \in L^X$, let

$$A_{[a]} = \{x \in X | A(x) \geq a\},$$

$$A_{(a)} = \{x \in X | a \in \beta(A(x))\},$$

$$A^{(a)} = \{x \in X | A(x) \not\leq a\}.$$

An L -topological space (L -space for short) is a pair (X, δ) , where δ is a subfamily of L^X which contains $\underline{0}$ $\underline{1}$ and is closed for any suprema and infima. δ is called an L -topology on X . Every member of δ is called an open L -set and its quasicomplementation is called a closed L -set.

If (X, τ) is a topological space, then $\omega(\tau)$ denote the L -topology of the continuous functions from (X, τ) to L with its Scott topology. $(X, \omega(\tau))$ is called to be topologically generated by (X, δ) .

For a subfamily $\Phi \subseteq L^X, 2^{(\Phi)}$ denotes the set of all finite subfamily of Φ .

Definition 2.1 ([1, 9]) Let (X, δ) be an L -space. $A \in L^X$ is called semiopen if and only if there exists $B \in \delta$ such that $B \leq A \leq cl(B)$. $A \in L^X$ is called semiclosed if and only if A' is semiopen.

Definition 2.2 ([1, 9]) Let (X, δ) be an L -space, $A \in L^X$. The set $cl_*A = \bigwedge \{B \in L^X | A \leq B \text{ and } B \text{ is semiclosed}\}$ is called the semiclosure of A . $int_*A = \bigvee \{B \in L^X | B \leq A \text{ and } B \text{ is semiopen}\}$ is called the semiinterior of A .

Definition 2.3 ([10]) Let (X, δ) be an L -space. $G \in L^X$ is called compact if for every family \mathcal{U} of open L -sets, it follows that

$$\bigwedge_{x \in X} (G'(x) \vee \bigvee_{A \in \mathcal{U}} A(x)) \leq \bigvee_{\mathcal{V} \in \langle \mathcal{U} \rangle} \bigwedge_{x \in X} (G'(x) \vee \bigvee_{A \in \mathcal{V}} A(x))$$

Lemma 2.4 ([10]) Let L be complete Heyting algebra and let $f : (X, \delta_1) \rightarrow (Y, \delta_2)$ be a mapping. Then for any family $\mathcal{P} \subset L^Y$

$$\bigvee_{y \in Y} (f(G)(y) \wedge \bigwedge_{B \in \mathcal{P}} B(y)) = \bigvee_{x \in X} (G(x) \wedge \bigwedge_{B \in \mathcal{P}} f^{-1}(B)(x))$$

Definition 2.5 ([10, 11]) Let (X, δ) be an L -space, $a \in L \setminus \{1\}$, and $G \in L^X$. A family $\mathcal{U} \subset L^X$ is said to be

- (1) an a -shading of G if for any $x \in X, (G'(x) \vee \bigvee_{A \in \mathcal{U}} A(x)) \not\leq a$;
- (2) a strong a -shading of G if $\bigwedge_{x \in X} (G'(x) \vee \bigvee_{A \in \mathcal{U}} A(x)) \not\leq a$;
- (3) an a -remote family \mathcal{P} of G if for any $x \in X, (G(x) \wedge \bigwedge_{A \in \mathcal{U}} B(x)) \not\leq a$;
- (4) a strong a -remote family of G if $\bigvee_{x \in X} (G(x) \wedge \bigwedge_{A \in \mathcal{U}} A(x)) \not\leq a$.

Definition 2.6 ([10]) Let $a \in L \setminus \{0\}, G \in L^X$. A family $\mathcal{U} \subset L^X$ is called

- (1) a β_a - cover of G if for all $x \in X$, it follows that $a \in \beta(G'(x) \vee \bigvee_{A \in \mathcal{U}} A(x))$;
- (2) a strong β_a - cover of G if $a \in \beta(\bigwedge_{x \in X} (G'(x) \vee \bigvee_{A \in \mathcal{U}} A(x)))$;
- (3) a Q_a - cover of G if for all $x \in X$, it follows that $G'(x) \vee \bigvee_{A \in \mathcal{U}} A(x) \geq a$.

Definition 2.7 ([1, 9]) Let (X, δ_1) and (Y, δ_2) be two L -spaces. $f : (X, \delta_1) \rightarrow (Y, \delta_2)$ is called

- (1) semicontinuous if $f^{-1}(A)$ is semiopen in (X, δ_1) for every open L -set A in (Y, δ_2) .
- (2) irresolute if $f^{-1}(A)$ is semiopen in (X, δ_1) for every semiopen L -set A in (Y, δ_2) .

3. Definition and characterizations of S^* closedness

Definition 3.1 Let (X, δ) be an L -space. $G \in L^X$ is called S^* closedness if for every family \mathcal{U} of semiopen L -sets, it follows that

$$\bigwedge_{x \in X} (G'(x) \vee \bigvee_{A \in \mathcal{U}} A(x)) \leq \bigvee_{\mathcal{V} \in (\mathcal{U})} \bigwedge_{x \in X} (G'(x) \vee \bigvee_{A \in \mathcal{V}} cl_* A(x))$$

Example 3.2 Suppose $X = \{x\}$, and $L = \{0, 1/4, 1/2, 3/4, 1\}$. For $a \in L$, $a' = 1 - a$. Let $\delta = \{\underline{0}, A, \underline{1}\}$, where $A(x) = 3/4$, then (X, δ) is an L -space. We can see that every L -set in (X, δ) is S^* closedness.

Since any open L -set must be semiopen L -set, we have the following theorem.

Theorem 3.3 Compactness is S^* closedness in L -space.

From Definition 3.1 and quasicomplementation of L , we can obtain the following theorem.

Theorem 3.4 Let (X, δ) be an L -space. $G \in L^X$ is S^* closedness if and only for every family \mathcal{B} of semiclosed L -sets, it follows that

$$\bigvee_{x \in X} (G(x) \wedge \bigwedge_{B \in \mathcal{B}} B(x)) \geq \bigwedge_{\mathcal{F} \in (\mathcal{B})} \bigvee_{x \in X} (G(x) \wedge \bigwedge_{B \in \mathcal{F}} int_* B(x))$$

Definition 3.5 Let $a \in L \setminus \{0\}$, $G \in L^X$. A family $\mathcal{U} \subset L^X$ is said to have weak a -nonempty intersection in G if $\bigvee_{x \in X} (G(x) \wedge \bigwedge_{A \in \mathcal{U}} A(x)) \geq a$. \mathcal{U} is said to have finite weak a_* -intersection property in G if for every finite

subfamily \mathcal{F} of \mathcal{U} , the family $\mathcal{F}_* = \{int_*F | F \in \mathcal{F}\}$ has weak a -nonempty intersection in G .

Theorem 3.6 Let (X, δ) be an L -space, $G \in L^X$. Then the following conditions are equivalent:

- (1) G is S^* closedness.
- (2) For any $a \in L \setminus \{1\}$, each semiopen strong a - shading \mathcal{U} of G has a finite subfamily \mathcal{V} such that $\{cl_*A | A \in \mathcal{V}\}$ is a strong a - shading of G .
- (3) For any $a \in L \setminus \{0\}$, each semiclosed strong a - remote family \mathcal{P} of G has a finite subfamily \mathcal{F} such that $\{int_*A | A \in \mathcal{P}\}$ is a strong a - remote family of G .
- (4) For any $a \in L \setminus \{0\}$, each family of semiclosed L -sets which has the finite weak a_* -intersection property in G has weak a -nonempty intersection in G .

Proof The proofs follow from Definition 3.1 and Theorem 3.4.

4. Properties of S^* closedness

Theorem 4.1 Let L be complete Heyting algebra. If G and H are S^* closedness, then so is $G \vee H$.

Proof For any family \mathcal{P} of of semiclosed L - sets, by Theorem 3.4 we have that

$$\begin{aligned}
 & \bigvee_{x \in X} ((G \vee H)(x) \wedge \bigwedge_{B \in \mathcal{P}} B(x)) \\
 &= \{ \bigvee_{x \in X} (G(x) \wedge \bigwedge_{B \in \mathcal{P}} B(x)) \} \vee \{ \bigvee_{x \in X} (H(x) \wedge \bigwedge_{B \in \mathcal{P}} B(x)) \} \\
 &\geq \{ \bigwedge_{\mathcal{F} \in \mathcal{C}(\mathcal{P})} \bigvee_{x \in X} (G(x) \wedge \bigwedge_{B \in \mathcal{F}} int_*B(x)) \} \vee \{ \bigwedge_{\mathcal{F} \in \mathcal{C}(\mathcal{P})} \bigvee_{x \in X} (H(x) \wedge \bigwedge_{B \in \mathcal{F}} int_*B(x)) \} \\
 &= \bigwedge_{\mathcal{F} \in \mathcal{C}(\mathcal{P})} \bigvee_{x \in X} ((G \vee H)(x) \wedge \bigwedge_{B \in \mathcal{F}} int_*B(x))
 \end{aligned}$$

This shows that $G \vee H$ is S^* closedness.

Theorem 4.2 If G is S^* closedness and H is a semiclosed set, then $G \wedge H$ is S^* closedness.

Proof For any family \mathcal{P} of of semiclosed L - sets, by Theorem 3.4 we have that

$$\begin{aligned}
& \bigvee_{x \in X} ((G \wedge H)(x) \wedge \bigwedge_{B \in \mathcal{P}} B(x)) \\
&= \bigvee_{x \in X} (G(x) \wedge \bigwedge_{B \in \mathcal{P} \cup \{\mathcal{H}\}} B(x)) \\
&\geq \{ \bigwedge_{\mathcal{F} \in \mathcal{C}(\mathcal{P} \cup \{\mathcal{H}\})} \bigvee_{x \in X} (G(x) \wedge \bigwedge_{B \in \mathcal{F}} \text{int}_* B(x)) \} \\
&= \{ \bigwedge_{\mathcal{F} \in \mathcal{C}(\mathcal{P})} \bigvee_{x \in X} (G(x) \wedge \bigwedge_{B \in \mathcal{F}} \text{int}_* B(x)) \} \\
&\bigvee \{ \bigwedge_{\mathcal{F} \in \mathcal{C}(\mathcal{P})} \bigvee_{x \in X} (G(x) \wedge H(x) \wedge \bigwedge_{B \in \mathcal{F}} \text{int}_* B(x)) \} \\
&= \bigwedge_{\mathcal{F} \in \mathcal{C}(\mathcal{P})} \bigvee_{x \in X} ((G \wedge H)(x) \wedge \bigwedge_{B \in \mathcal{F}} \text{int}_* B(x))
\end{aligned}$$

Then we obtain that $G \wedge H$ is S^* closedness.

Theorem 4.3 Let L be complete Heyting algebra and $f : (X, \delta_1) \rightarrow (Y, \delta_2)$ be an irresolute mapping. If G is S^* closedness in (X, δ_1) , then $f(G)$ is S^* closedness in (Y, δ_2) .

Proof For any family \mathcal{P} of of semiclosed L - sets in (Y, δ_2) , by Lemma 2.4 and S^* closedness of G we have that

$$\begin{aligned}
& \bigvee_{y \in Y} ((f(G))(y) \wedge \bigwedge_{B \in \mathcal{P}} B(y)) \\
&= \bigvee_{x \in X} (G(x) \wedge \bigwedge_{B \in \mathcal{P}} f^{-1}(B)(x)) \\
&\geq \bigwedge_{\mathcal{F} \in \mathcal{C}(\mathcal{P})} \bigvee_{x \in X} (G(x) \wedge \bigwedge_{B \in \mathcal{F}} \text{int}_* f^{-1}(B)(x)) \\
&\geq \bigwedge_{\mathcal{F} \in \mathcal{C}(\mathcal{P})} \bigvee_{x \in X} (G(x) \wedge \bigwedge_{B \in \mathcal{F}} f^{-1}(\text{int}_* B)(x)) \\
&= \bigwedge_{\mathcal{F} \in \mathcal{C}(\mathcal{P})} \bigvee_{y \in Y} (f(G)(y) \wedge \bigwedge_{B \in \mathcal{F}} \text{int}_* B(y))
\end{aligned}$$

Hence $f(G)$ is S^* closedness.

Analogously, we can obtain the following theorem.

Theorem 4.4 Let L be complete Heyting algebra and $f : (X, \delta_1) \rightarrow (Y, \delta_2)$ be a semicontinuous mapping. If G is S^* closedness in (X, δ_1) , then $f(G)$ is S^* closedness in (Y, δ_2) .

5. Further characterizations of S^* closedness and goodness

In this section, we assume that L is a completely distributive *DeMorgan* algebra.

Theorem 5.1 Let (X, δ) be an L -space, $G \in L^X$. Then the following statements are equivalent:

- (1) G is S^* closedness.
- (2) For any $a \in L \setminus \{0\}$, each semiclosed strong a - remote family \mathcal{P} of G has a finite subfamily \mathcal{F} such that $\{int_* A | A \in \mathcal{F}\}$ is a strong a - remote family of G .
- (3) For any $a \in L \setminus \{0\}$, each semiclosed strong a - remote family \mathcal{P} of G has a finite subfamily \mathcal{F} such that $\{int_* A | A \in \mathcal{F}\}$ is an a - remote family of G .
- (4) For any $a \in L \setminus \{0\}$, each semiclosed strong a - remote family \mathcal{P} of G there exist a finite subfamily \mathcal{F} of \mathcal{P} and $b \in \beta(a)$ such that $\{int_* A | A \in \mathcal{F}\}$ is a strong b - remote family of G .
- (5) For any $a \in L \setminus \{0\}$, each semiclosed strong a - remote family \mathcal{P} of G there exist a finite subfamily \mathcal{F} of \mathcal{P} and $b \in \beta(a)$ such that $\{int_* A | A \in \mathcal{F}\}$ is a b - remote family of G .
- (6) For any $a \in L \setminus \{1\}$, each semiopen strong a - shading \mathcal{U} of G has a finite subfamily \mathcal{V} such that $\{cl_* A | A \in \mathcal{V}\}$ is a strong a - shading of G .
- (7) For any $a \in L \setminus \{1\}$, each semiopen strong a - shading \mathcal{U} of G has a finite subfamily \mathcal{V} such that $\{cl_* A | A \in \mathcal{V}\}$ is an a - shading of G .
- (8) For any $a \in L \setminus \{1\}$ and any semiopen strong a - shading \mathcal{U} of G , there exist a finite subfamily \mathcal{V} and $b \in \alpha(a)$ such that $\{cl_* A | A \in \mathcal{V}\}$ is a b - shading of G .
- (9) For any $a \in L \setminus \{1\}$ and any semiopen strong a - shading \mathcal{U} of G , there exist a finite subfamily \mathcal{V} and $b \in \alpha(a)$ such that $\{cl_* A | A \in \mathcal{V}\}$ is a strong b - shading of G .
- (10) For any $a \in L \setminus \{0\}$, each semiopen strong β_a -cover \mathcal{U} of G has a finite subfamily \mathcal{V} such that $\{cl_* A | A \in \mathcal{V}\}$ is a strong β_a -cover of G .
- (11) For any $a \in L \setminus \{0\}$, each semiopen strong β_a -cover \mathcal{U} of G has a finite subfamily \mathcal{V} such that $\{cl_* A | A \in \mathcal{V}\}$ is a β_a -cover of G .
- (12) For any $a \in L \setminus \{0\}$ and any semiopen strong β_a -cover \mathcal{U} of G , there exist a finite subfamily \mathcal{V} and $b \in L$ with $a \in \beta(b)$ such that $\{cl_* A | A \in \mathcal{V}\}$ is a strong β_a -cover of G .
- (13) For any $a \in L \setminus \{0\}$ and any semiopen strong β_a -cover \mathcal{U} of G , there exist a finite subfamily \mathcal{V} and $b \in L$ with $a \in \beta(b)$ such that $\{cl_* A | A \in \mathcal{V}\}$ is a β_a -cover of G .

(14) For any $a \in L \setminus \{0\}$ and $b \in \beta(a) \setminus \{0\}$, each semiopen Q_a -cover \mathcal{U} of G has a finite subfamily \mathcal{V} such that $\{cl_*A | A \in \mathcal{V}\}$ is a Q_b -cover of G .

(15) For any $a \in L \setminus \{0\}$ and $b \in \beta(a) \setminus \{0\}$, each semiopen Q_a -cover \mathcal{U} of G has a finite subfamily \mathcal{V} such that $\{cl_*A | A \in \mathcal{V}\}$ is a strong β_b -cover of G .

(16) For any $a \in L \setminus \{0\}$ and $b \in \beta(a) \setminus \{0\}$, each semiopen Q_a -cover \mathcal{U} of G has a finite subfamily \mathcal{V} such that $\{cl_*A | A \in \mathcal{V}\}$ is a β_b -cover of G .

Proof (1) \Leftrightarrow (2) It follows from Theorem 3.6.

(2) \Rightarrow (3) It is easy for the fact that every strong a - remote family of G is a - remote family of G .

(3) \Rightarrow (4) Let $a \in L \setminus \{0\}$. Suppose \mathcal{U} is semiclosed strong a - remote family of G , then $\bigvee_{x \in X} (G(x) \wedge \bigwedge_{A \in \mathcal{U}} A(x)) \not\leq a$. Take $c \in \beta(a)$ such that $\bigvee_{x \in X} (G(x) \wedge \bigwedge_{A \in \mathcal{U}} A(x)) \not\leq c$. One can see that \mathcal{U} is semiclosed strong c - remote family of G . From (3), \mathcal{U} has a finite subfamily \mathcal{F} such that $\{int_*A | A \in \mathcal{F}\}$ is c - remote family of G . Choose $b \in \beta(a)$ such that $c \in \beta(b)$, then $\{int_*A | A \in \mathcal{F}\}$ is strong b - remote family of G .

(4) \Rightarrow (5) \Rightarrow (2) It is Obvious.

(1) \Leftrightarrow (6) It follows from Theorem 3.6.

(6) \Rightarrow (7) It is easy for the fact that every strong a -shading family of G is a -shading family of G .

(7) \Rightarrow (8) Let $a \in L \setminus \{1\}$. Suppose \mathcal{U} is semiclosed strong a - shading family of G , then $\bigwedge_{x \in X} (G'(x) \vee \bigvee_{A \in \mathcal{U}} A(x)) \not\leq a$. Take $c \in \alpha(a)$ such that $\bigwedge_{x \in X} (G'(x) \vee \bigvee_{A \in \mathcal{U}} A(x)) \not\leq c$. One can see that \mathcal{U} is semiclosed strong c -shading family of G . From (7), \mathcal{U} has a finite subfamily \mathcal{F} such that $\{int_*A | A \in \mathcal{F}\}$ is c -shading family of G . Choose $b \in \alpha(a)$ such that $c \in \alpha(b)$, then $\{int_*A | A \in \mathcal{F}\}$ is strong b -shading family of G .

(8) \Rightarrow (9) \Rightarrow (6) It is Obvious.

Similarly we can prove the other statements.

Remark 5.2 In Theorem 5.1, $a \in L \setminus \{0\}$ and $b \in \beta(a)$ can be replaced by $a \in M(L)$ and $b \in \beta^*(a)$ respectively. $a \in L \setminus \{1\}$ and $b \in \alpha(a)$ can be replaced by $a \in P(L)$ and $b \in \alpha^*(a)$ respectively. Thus, we can obtain other 15 equivalent conditions of the S^* closedness.

Lemma 5.3([11]) Let $(X, \omega(\delta))$ be generated topologically by (X, δ) . If A is a semiopen set in (X, δ) , then χ_A is a semiopen set L - set in $(X, \omega(\delta))$. If B is a semiopen set in $(X, \omega(\delta))$, then $B_{(a)}$ is a semiopen set in (X, δ) .

Lemma 5.4 Let $(X, \omega(\delta))$ be generated topologically by $(X, \delta), B \in L^X$. Then $cl_*(B_{[b]}) \subset (cl_*B)_{[b]}$ in $(X, \omega(\delta))$.

Proof For $B \in L^X, B_{[b]} \subset (cl_*B)_{[b]}$. Then $cl_*(B_{[b]}) \subset (cl_*B)_{[b]}$ in $(X, \omega(\delta))$ since $(cl_*B)_{[b]}$ is a semiclosed set in $(X, \omega(\delta))$ and $cl_*(B_{[b]})$ is the smallest semiclosed set contains $B_{[b]}$.

Theorem 5.5 Let (X, δ) be a topological space and $(X, \omega(\delta))$ be generated topologically by (X, δ) . Then $(X, \omega(\delta))$ is S^* closedness if and only if (X, δ) is S^* closedness.

Proof Let \mathcal{A} be a semiopen-cover of (X, δ) . Then $\{\chi_A | A \in \mathcal{A}\}$ is a family of semiopen L - sets in $(X, \omega(\delta))$ with $\bigwedge_{x \in X} (\bigvee_{A \in \mathcal{U}} \chi_A(x)) = \underline{1}$. From the S^* closedness of $(X, \omega(\delta))$, it follows that

$$\underline{1} = \bigvee_{\mathcal{V} \in \in(\mathcal{U})} \bigwedge_{x \in X} (\bigvee_{A \in \mathcal{V}} cl_*(\chi_A)(x)) \leq \bigvee_{\mathcal{V} \in \in(\mathcal{U})} \bigwedge_{x \in X} (\bigvee_{A \in \mathcal{V}} \chi_{cl_*A}(x))$$

This implies that there exists $\mathcal{V} \in \in(\mathcal{U})$ such that $\bigvee_{A \in \mathcal{V}} \chi_{cl_*A}(x) = \underline{1}$.

Hence, the family of $\{cl_*(A) | A \in \mathcal{V}\}$ is a cover of (X, δ) . Then (X, δ) is S^* closedness.

Conversely, let \mathcal{U} be a family of semiopen L - sets in $(X, \omega(\delta))$ and let $\bigwedge_{x \in X} (\bigvee_{B \in \mathcal{U}} B(x)) = a$. If $a = 0$, then obviously

$$\bigwedge_{x \in X} (\bigvee_{B \in \mathcal{U}} B(x)) \leq \bigvee_{\mathcal{V} \in \in(\mathcal{U})} \bigwedge_{x \in X} (\bigvee_{B \in \mathcal{V}} cl_*B(x))$$

Now we suppose that $a \neq 0$. In this case, for any $b \in \beta(a) \setminus \{0\}$, we have that

$$b \in \beta(\bigwedge_{x \in X} (\bigvee_{B \in \mathcal{U}} B(x))) \subset \bigcap_{x \in X} \beta(\bigvee_{B \in \mathcal{U}} B(x)) = \bigcap_{x \in X} \bigcup_{B \in \mathcal{U}} \beta(B(x))$$

This implies the family of $\{B_{(b)} | B \in \mathcal{U}\}$ is a semiopen cover of (X, δ) by Lemma 5.3, From the S^* closedness of (X, δ) , there exists $\mathcal{V} \in \in(\mathcal{U})$ such that $\{cl_*(B_{(b)}) | B \in \mathcal{V}\}$ is a cover of (X, δ) . Obviously $\{(cl_*B)_{[b]} | B \in \mathcal{V}\}$ is a cover of (X, δ) since $cl_*(B_{(b)}) \subset cl_*(B_{[b]}) \subset (cl_*B)_{[b]}$. Hence $b \leq \bigwedge_{x \in X} (\bigvee_{B \in \mathcal{V}} cl_*B(x))$. Further we have that

$$b \leq \bigwedge_{x \in X} (\bigvee_{B \in \mathcal{V}} cl_*B(x)) \leq \bigvee_{\mathcal{V} \in \in(\mathcal{U})} \bigwedge_{x \in X} (\bigvee_{B \in \mathcal{V}} cl_*B(x))$$

This implies that

$$\bigwedge_{x \in X} (\bigvee_{B \in \mathcal{U}} B(x)) = a = \bigvee \{b | b \in \beta(a)\} \leq \bigvee_{\mathcal{V} \in \in(\mathcal{U})} \bigwedge_{x \in X} (\bigvee_{B \in \mathcal{V}} cl_*B(x))$$

Therefore $(X, \omega(\delta))$ is S^* closedness.

Corollary 5.6 S^* closedness in L -topological spaces is good extension.

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