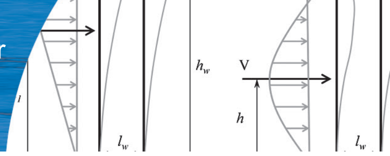


Simple estimation of the maximum elastic roof displacement of a slender cantilever RC wall accounting for dynamic effects

Estimación simple del desplazamiento elástico máximo de techo de un muro esbelto de hormigón armado en voladizo considerando efectos dinámicos

Fecha de entrega: 18 de octubre 2017
Fecha de aceptación: 8 de marzo 2018

Patricio Quintana Gallo

Escuela de Ingeniería Civil, Universidad de Valparaíso, General Cruz 222, Piso 4, Valparaíso, Chile, patricio.quintana@uv.cl

This article presents a simple method for estimating the maximum elastic roof displacement of a slender cantilever reinforced concrete RC wall, accounting for dynamic effects, named δ^v_{te}. The formulation computes δ^v_{te} as a function of an equivalent concentrated lateral load, acting at an equivalent height h_v. The dynamic effects are included by calculating the equivalent height of a load pattern representative of the first mode of vibration, h_1, and reducing it to be consistent with a lateral load distribution that imposes a deformed shape representative of higher modes upon the wall. This is executed when dividing h_1 by the dynamic amplification factor ω_v, previously defined for capacity-based shear design. The displacement δ^v_{te} is obtained by imposing nominal yielding conditions at the critical cross-section of the wall, for the lateral load acting at the reduced height h_v. Including well-established expressions for the nominal yielding curvature of RC cross-sections, a new formula for computing the maximum elastic top lateral drift ratio of the wall as a function of dimensionless numbers associated to the wall geometry, topology, and reinforcing steel is proposed. Using an example, it is shown that the novel expression provides more conservative results compared to those obtained with classical and recently proposed formulas, noting that this results into larger extensions of horizontal boundary confinement elements of a wall, for the same ultimate roof displacement. To conclude, the formulation is presented in a way suitable for its implementation within the Chilean code, and in simplified versions useful for quick hand calculations.

Keywords: reinforced concrete RC, slender cantilever wall, lateral roof displacement, elastic limit, dynamic effects

Este artículo presenta un método simple para estimar el desplazamiento elástico máximo de techo de un muro esbelto de hormigón armado en voladizo, incluyendo efectos dinámicos, llamado δ^v_{te}. La formulación calcula δ^v_{te} en función de una fuerza lateral equivalente, ubicada en una altura equivalente h_v. Los efectos dinámicos son incluidos calculando la altura equivalente asociada a una distribución de cargas laterales representativa del primer modo, h_1, reduciéndola para ser consistente con un patrón de cargas laterales que impone una deformada representativa de modos superiores de vibrar sobre el muro. Esto se ejecuta dividiendo h_1 por el factor de amplificación dinámica ω_v, definido previamente para diseño al corte por capacidad. El desplazamiento δ^v_{te} se obtiene al imponer condiciones de fluencia nominal en la sección crítica del muro, para la carga equivalente aplicada en h_v. Añadiendo expresiones bien establecidas para calcular la curvatura nominal de fluencia de secciones transversales de miembros de hormigón armado, se presenta una nueva fórmula para calcular la máxima razón de desplazamiento elástico de techo, en función de números adimensionales que dependen de la geometría, topología, y acero de refuerzo del muro. Usando un ejemplo, se demuestra que el nuevo método provee valores más conservadores que otros propuestos anteriormente, notando que esto resulta en mayores extensiones horizontales de elementos de borde, para el mismo desplazamiento último de techo. Para concluir, la formulación se presenta en una forma apta para ser implementada en la regulación sísmica chilena, y en versiones simplificadas útiles para cálculos a mano.

Palabras claves: hormigón armado, muro esbelto en voladizo, desplazamiento lateral de techo, límite elástico, efectos dinámicos

Introduction

The seismic demands placed upon reinforced concrete RC walls can be visualized as a set of equivalent lateral loads with not necessarily equal magnitude at every floor level. The trivial case of this is a uniform distribution. To evaluate the maximum elastic (or nominal yielding) displacement of a cantilever wall at the roof level, the so called inverted triangle distribution is typically used. This corresponds to a linear variation of the lateral load with a maximum at the roof (top) level, and equal to zero at the base (*e.g.* Wallace and Moehle, 1992). Alternatively, it is often assumed that all the equivalent lateral load is concentrated at the roof level, as in direct displacement based design procedures (Priestley and Kowalsky, 1998; Paulay, 2002; Priestley *et al*., 2007), and previous design guides (Park and Paulay, 1975, Paulay and Priestley, 1993).

The load patterns mentioned in the previous paragraph are aimed at imposing a deflected shape representative of the first mode of vibration of the wall. Nevertheless, it is acknowledged that the actual distribution of equivalent lateral forces along the height of the wall varies in time, and depends on the relative predominance of the modes of vibration and their periods, not only the first one. The consequence is a possible overestimation of the maximum elastic roof displacement of cantilever walls, generically named δ_{te}, when calculated per the aforementioned assumptions.

In the following, a simple method built upon the analogy with a cantilever beam loaded with a concentrated vertical force at a given distance from the support is presented. The deflection at the top of a cantilever wall within the elastic range, generically named δ_t, is calculated as the result of the action of a lumped lateral load V, placed at a height h from the critical section, such that it produces the same reactions at the base of the wall, compared to a distributed load pattern. Firstly, h is calculated for the first mode of vibration, such that $h = h_1$. Subsequently, the formulation incorporates a reduction of h_1 via ω_v, a dynamic amplification factor (Paulay and Priestley, 1992; Priestley *et al*., 2007; Rutenberg, 2013), to account for dynamic effects in the calculation of δ_t, as initially proposed by Paulay and Priestley (1992) for estimating the shear demands placed upon RC walls during earthquakes, following capacity design principles. Finally, by imposing yielding conditions at the critical section of the wall, such that $\delta_{te} = \delta^v_{te}$, the maximum elastic roof displacement that accounts for dynamic effects, is calculated.

It is shown that the proposed formulae provide a more conservative approach for estimating δ_{te}, compared to others proposed in the literature (*e.g.* Wallace and Moehle, 1992; Priestley *et al*., 2007; Massone *et al*., 2015).

Equivalent lateral load location, dynamic effects and shear demands

In the approach introduced in this article, the lateral load pattern corresponding to the equivalent lateral seismic actions along the height of the wall, is represented by an equivalent concentrated lateral load V. Per equilibrium, this force is equal to the shear at the base of the wall, and is located at h = M/V, where M is the overturning moment at the base of the wall (the critical section in this case) produced by V. Figure 1 presents two scenarios for this location: (1) V = V_1 and $h = h_1$, the lateral force and its location associated to the first mode; and (2) V = V_v and $h = h_v$, the lateral force and its location which account for the dynamic effects produced by the higher modes.

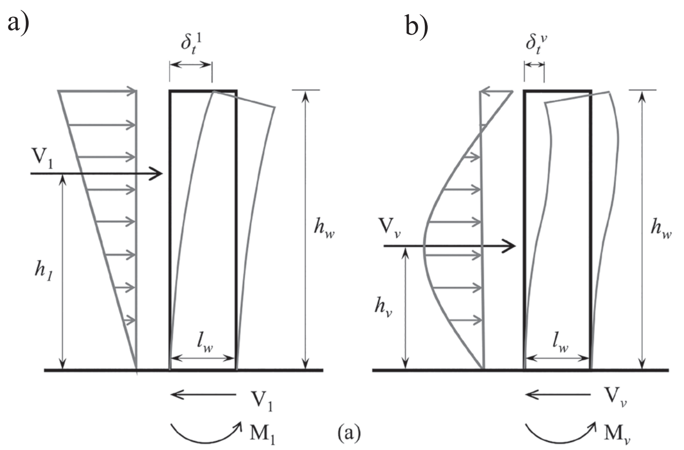
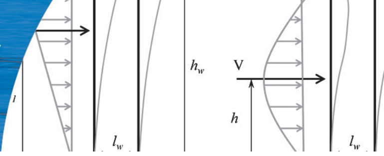


Figure 1: Equivalent lateral load and its location: a) inverted triangle distribution, first mode deflected shape and b) higher-mode load distribution and deflected shape

Figure 1a shows the equilibrium scenario of a slender cantilever wall subjected to an inverted triangle load distribution, associated to the first mode of vibration. In this case, the position of the equivalent load V_1, is $h_1 = 2/3h_w$, by definition. In the limit at the onset of the nominal yielding, *i.e.* when $M_1 = M_y$ and the curvature $\phi = \phi_y$ at the



base of the wall, the base shear and equivalent lateral load is well determined by (1), such that:

$$V_1 = M_1/h_1 = M_y/h_1 = \tfrac{3}{2} M_y/h_w \quad (1)$$

The resulting δ_t associated to this scenario is named δ_t^1, where the superscript '1' refer to the first mode. When yielding is reached at the critical section of the wall, this displacement is referred to as the maximum elastic displacement, and is named δ_{te}^1. This displacement is obtained by double integration of the curvature diagram over h_w, assuming EI constant (Wallace and Moehle, 1992) and is given by (2):

$$\delta_{te}^1 = \tfrac{11}{40} \phi_y h_w^2 \quad (2)$$

Figure 1b shows a different situation, where the load distribution is not fully determined, but it imposes a deflected shape representative of higher-modes of vibration upon the wall, such that the equivalent lateral force is V_v. If in this case the elastic limit at the base of the wall is also imposed, $M_v = M_y$, with M_v the bending moment at the base of the wall due to the higher-mode load pattern. As depicted in Figure 1, h_v is smaller than h_1, such that $h_v = h_1/\omega_v$, with $\omega_v > 1.0$, the dynamic amplification factor.

One consequence of the above is that the base shear V_v, depends not only on M_y and h_w, but also on ω_v, such that:

$$V_v = M_y/(h_1/\omega_v) = \tfrac{3}{2}\omega_v M_y/h_w = \omega_v V_1 \quad (3)$$

This is the principle behind the capacity-based procedure for shear design originally proposed by Paulay and Priestley in 1992. Nevertheless, as this method focuses on an ultimate limit state, the over-strength of the resisting moment at the base of the wall must be included, *e.g.* via $\Omega_o = \lambda M_n/M_y$, where the probable resisting moment at the base of the wall is defined as the nominal resisting moment M_n times a factor $\lambda > 1.0$ that accounts for hardening, and an increased yielding stress of the reinforcing steel with respect to the nominal value. In this situation, the capacity-based shear demand is given by (4):

$$V_{v,0} = \Omega_o \omega_v V_1 \quad (4)$$

a well-established formula since 1992 (Paulay and Priestley, 1992; SNZ 3101, 2006). It is important to recall that the first-mode reference force V_1 is equal to the base shear resulting from the application of the lateral load distribution shown in Figure 1a, or, alternatively, of the equivalent lateral loads prescribed by codes, as required by SNZ 3101 (2006), for example. In any case, V_1 should not be the shear obtained with a modal spectral analysis and a certain modal combination, because it already includes the effect of the higher modes, in a different way. Hence, this effect would be doubled when including the factor ω_v in the procedure.

A second implication of $h_v < h_1$, as explained later, is that $\delta_{te}^v < \delta_{te}^1$. An analytical expression for δ_{te}^v defined for the situation depicted in Figure 1b to evaluate differences with δ_{te}^1, however, had not been proposed yet. This can be partially attributed to the lack of an *ad hoc* intuitive lateral load distribution representative of that shown in Figure 1b (such as the inverted triangle for the first mode), which would allow obtaining δ_{te}^v by double integration of the curvature diagram along the height of the wall, as for obtaining (2). This could also be partially due to the belief that the elastic limit at the base of the wall can only be reached with a deformed shape representative of the first mode, as typically the maximum inelastic response of the wall is assumed to occur whilst it deforms in this situation (*e.g.* in Priestley *et al.*, 2007).

In the following, an analytical expression for calculating δ_{te}^v, which uses the conceptual features of the dynamic amplification factor ω_v is developed.

Lateral roof displacement including dynamic effects

Consider a cantilever beam of length L and constant stiffness EI, loaded with a vertical force F located at a distance a from the fixed end, as shown in Figure 2. As can be determined using energy methods, described in most structural analysis textbooks, the vertical displacement Δ at the free end of the cantilever beam shown in Figure 2, considering flexural deformations only, is given by (5):

$$\Delta = \int_0^a \frac{F(x-a)(x-L)}{EI} dx = \frac{Fa^2}{6EI}(3L - a) \quad (5)$$

Similarly, referring to Figure 1b, the lateral displacement at the top of a slender cantilever wall in the elastic range, δ_t^v, produced by the lateral load V_v, acting at the height h_v

is:

$$\delta_t^v = \frac{V_v h_v^2}{6EI}(3h_w - h_v) \quad (6)$$

where, EI is the flexural stiffness of the cross-section of the wall (moment-curvature stiffness). Note that the right hand side of (5) is valid only if EI is constant along the height of the wall. In this case, as a first approximation, whose limitations are included at the end of this article, EI is taken constant along h_w, and equal to that of the critical cross-section of the wall, as explained below.

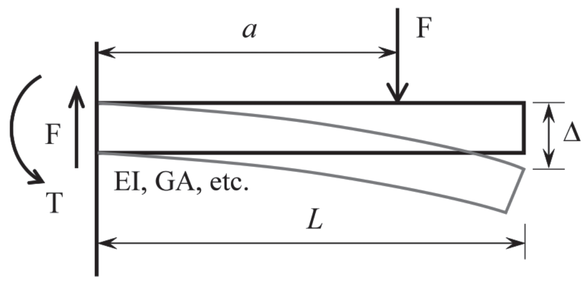


Figure 2: Cantilever beam analogy

With reference to Figure 1b, by equilibrium, $M_v = V_v h_v$. Up to the yielding point, in the elastic range, this moment is linearly proportional to the curvature of the crossed-section at the base of the wall ϕ, such that $M_v = EI\phi$. Equating these two expressions and rearranging, it follows that:

$$V_v = \frac{EI\phi}{h_v} \quad (7)$$

Replacing (7) into (6), leads to (8):

$$\delta_t^v = \frac{\phi h_v}{6}(3h_w - h_v) \quad (8)$$

In the elastic limit or yielding point, the curvature at the base of the wall is $\phi = \phi_y$, the nominal yielding curvature. This curvature is further defined (Paulay, 2002) as in (9) and (10):

$$\phi_y = (M_y/M'_y)\phi'_y = \frac{(M_y/M'_y)\varepsilon_y}{(\beta l_w)} \quad (9)$$

$$\phi_y = \eta(\varepsilon_y/l_w) \quad (10)$$

where, M'_y, ϕ'_y and β, are the bending moment, curvature, and neutral axis depth (as a fraction of l_w), respectively, at first yield; and ε_y the yielding strain of the reinforcing steel.

Combining (10) and (9), an expression for η takes the following form:

$$\eta = M_y \phi_y' l_w / (M'_y \varepsilon_y) \quad (11)$$

It has been shown (Priestley and Kowalsky, 1998; Paulay, 2002; Priestley, 2003) that η is approximately constant, and can be taken as $\eta = 2.0$ for rectangular reinforced concrete walls, for example, within a plus minus 15% error (Priestley, 2003). Nevertheless, it is easy to calculate this number on a case by case basis using a sectional analysis and (11).

Imposing $\phi = \phi_y$ in (8), such that $\delta_t^v = \delta_{te}^v$, and using expression (10) for ϕ_y, (8) becomes (12):

$$\delta_{te}^v = \frac{\eta \varepsilon_y h_v}{6 l_w}(3h_w - h_v) \quad (12)$$

Defining $\alpha = h_1/h_w$, the normalized height of the equivalent first-mode lateral load pattern, and recalling that $h_v = h_1/\omega_v$, (12) can be rewritten as:

$$\delta_{te}^v = \frac{\eta \varepsilon_y \alpha h_w}{6 l_w \omega_v}\left(3h_w - \frac{\alpha h_w}{\omega_v}\right) \quad (13)$$

Dividing (13) into h_w, and rearranging, it becomes:

$$\frac{\delta_{te}^v}{h_w} = (\alpha/6\omega_v)(3 - \alpha/\omega_v)\, \eta\, \varepsilon_y \left(\frac{h_w}{l_w}\right) \quad (14)$$

Further, defining $A_r = h_w/l_w$, the aspect ratio of the wall; and $dr_{te}^v = \delta_{te}^v/h_w$, the maximum elastic roof drift ratio including dynamic effects, (14) can be written as:

$$dr_{te}^v = (\alpha/6\omega_v)(3 - \alpha/\omega_v)\, \eta\, \varepsilon_y A_r \quad (15)$$

(15) is a novel expression that defines the maximum elastic roof displacement capacity of a slender cantilever wall, accounting for dynamic effects, in dimensionless terms. Obviously, when $\omega_v = 1.0$, (15) reduces to the case where $h_v = h_1$, and $dr_{te}^v = dr_{te}^1$, defined in (16):

$$dr_{te}^1 = (\alpha/6)(3 - \alpha)\, \eta\, \varepsilon_y A_r \quad (16)$$

To construct an expression for comparison with other non-dimensionless formulas with the form of (2), (14) is firstly divided by h_w, such that:

$$\frac{\delta_{te}^v}{h_w^2} = (\alpha/6\omega_v)(3 - \alpha/\omega_v)\left(\frac{\eta\, \varepsilon_y}{l_w}\right) \quad (17)$$

Noting that the third factor of the right-hand side of (17) is equal to ϕ_y as shown in (10), it follows that:

$$\delta^v_{te}\phi_y h_w^2 = (\alpha/6\omega_v)(3\text{-}\alpha/\omega_v) \qquad (18)$$

Defining $\gamma = \delta^v_{te}/(\phi_y h_w^2)$, and replacing into (18) yields:

$$\gamma = (\alpha/6\omega_v)(3\text{-}\alpha/\omega_v) \qquad (19)$$

The parameter γ defined in (19) serves for comparison with (2), where $\gamma = 11/40$, as well as with other limits proposed by other researchers, as shown later on.

Selection of ω_v

The variable ω_v depends on the height of the wall, or, indirectly, on the number of storeys of the building, as proposed by Paulay and Priestley (1992). There are several other expressions for ω_v. For a comprehensive review of the literature on this subject, the reader is referred to Rutenberg (2013).

In the formulation of Paulay and Priestley (1992) considered herein, ω_v varies linearly from 1.0 to 1.8 for one and six storeys, and it is limited to 1.8 for buildings of six storeys or more. Nevertheless, based on numerical work done by Quintana Gallo (2008), it is suggested that the upper limit of $\omega_v = 2.0$ be considered. This value is used in the formulation of the simplified expressions presented at the end of this article. On the other hand, the factor ω_v is explicitly included in the general formulation for δ^v_{te} to allow for the use of *ad hoc* amplification factors if desired.

Example for discussion

As an example for discussion, consider the case of a rectangular cantilever wall with the following properties: $h_w = 25$ m, and $l_w = 5$ m, such that $A_r = 5$. Take $\eta = 2.0$ for a rectangular wall (Priestley and Kowalsky, 1998; Paulay, 2002; Priestley *et al*., 2007), and consider a steel with $\varepsilon_y = 0.002 = 0.2\%$, as in the Chilean practice. Assume the action of an inverted triangle lateral load distribution (see Figure 1a), such that $\alpha = 2/3$. For now, neglect the dynamic effects, *i.e.* take $\omega_v = 1.0$.

Using the novel expression proposed in (19), $\gamma = 7/27 \approx 0.26$. From the classical expression presented in (2), $\gamma = 11/40 = 0.275$. Hence, expression (19) very closely approximates the elastic displacement obtained with (2) (Wallace and Moehle 1992). Now consider $\alpha = 1$, such that the equivalent lateral load is located at the roof level. In this case, (19) gives $\gamma = 1/3 \approx 0.33$, the value used in direct displacement based design (Priestley *et al*., 1998; Paulay, 2002) to estimate the yielding roof displacement of a wall, as initially proposed by Park and Paulay (1975) for cantilever beams.

Now include the dynamic effects. Note that for an inter-storey height, $h_s = 3$ m, typical of the New Zealand construction practice, the equivalent number of storeys of the wall of the example is $n = 25/3 \approx 8.3 > 6$. Note that in Chile h_s is typically equal to 2.6 m, such that n in this case, would be larger than its New Zealand counterpart. Hence, the upper limit of ω_v applies in both cases, and is herein conservatively taken as $\omega_v = 2.0$, as mentioned before. Replacing this and the other data into (19), $\gamma = 4/27 \approx 0.15$, which is significantly smaller than the previously examined values. This approximation is also more conservative than that proposed by Massone *et al.* (2015), for example, who suggest using $\gamma = 0.22$, based on the results of dynamic analyses.

Calculating dr_{te} for the wall of the example with (16), without consideration of dynamic effects, *i.e.* for the first mode only: $dr^1_{te} = (1/9)\cdot(7/3)\cdot 2.0\cdot 0.002\cdot 5 = (14/27)\% = 0.52\%$, a result almost identical to that obtained with the classical (2), divided by h_w. On the other hand, considering $\omega_v = 2.0$ and (15): $dr^v_{te} = (1/18)\cdot(8/3)\cdot 2.0\cdot 0.002\cdot 5 = (8/27)\% = 0.30\%$. Again, it is noted that neglecting the dynamic effects implies a significant overestimation of δ_{te}.

The estimation of δ_{te} is important in the design and detailing of confinement boundary elements of RC walls, within a plastic-hinge model approach, currently required by the Chilean RC code provisions (DS60, 2011). The reason is that an overestimation of δ_{te} leads to a smaller required plastic roof displacement δ_{tp} (and equivalently smaller plastic rotations at the base of the wall), for achieving the same ultimate roof lateral displacement δ_{tu}. As a result, smaller horizontal extensions of the boundary confinement elements would be required. Therefore, the approach introduced in this article might serve as a more conservative, yet rational, tool for design.

Complete and simplified proposed expressions

To be considered within the Chilean code requirements, the following expressions are suggested for computing δ^v_{te}. For the sake of simplicity, in the following, the scripts v and t are dropped, such that $\delta_e = \delta^v_{te}$, to be consistent with the nomenclature of DS60 (2011), such that:

$$\frac{\delta_e}{h_w} = (\alpha/6\omega_v)(3-\alpha/\omega_v)\,\eta\,\varepsilon_y \mathrm{A}_r \qquad (20)$$

with

$$\alpha = h_1/h_w \qquad (21)$$

and

$$h_1 = \textstyle\sum_{k=1}^{n} h_{1,k}F_{1,k}/\sum_{k=1}^{n} F_{1,k} \qquad (22)$$

where, $F_{1,k}$ and $h_{1,k}$ are the magnitude and height of the lateral force associated to the storey k (k = 1 to n), obtained with a code-prescribed equivalent lateral force analysis, *e.g.* that required by the Chilean standard NCh433 (INN, 2009).

Alternatively, assuming an inverted triangle load pattern, α = 2/3. Replacing this value in (20) yields:

$$\frac{\delta_e}{h_w} = \left[\frac{9\omega_v-2}{27\omega_v^2}\right]\eta\,\varepsilon_y \mathrm{A}_r \qquad (23)$$

In both formulations, the parameter η can be calculated using (11), or can be taken as:

η = 2.0 for rectangular and asymmetric (flanged) walls with the flange in tension,

η = 1.5 for asymmetric (flanged) walls with the flange in compression.

Further simplification of (23) by taking ω_v = 2.0, it reduces to (24):

$$\frac{\delta_e}{h_w} = \frac{4}{27}\eta\,\varepsilon_y \mathrm{A}_r \qquad (24)$$

Note that (24) is appropriate for a single degree of freedom (SDOF) system, where α = 1/3 and ω_v = 1.0, by definition. Replacing these values into (20) also leads to (24). Hence, for SDOF systems, (24) should be the equation to refer to.

As a rule of thumb, for rectangular walls, (24) can be additionally simplified taking η = 2.0, and ε_y = 0.2%, as in the Chilean practice, such that:

$$\frac{\delta_e}{h_w} = \frac{16}{27000}\mathrm{A}_r \approx 0.06\mathrm{A}_r(\%) \qquad (25)$$

For walls with flanged cross-sections with the flange acting in compression, the right hand side of (25) should be multiplied by 3/4, as in that case η = 1.5 instead of 2.0 (Priestley *et al.*, 2007; Quintana Gallo, 2008, 2014).

Finally, note that if one neglects the dynamic effects (*i.e.* ω_v = 1.0), as in a pushover analysis, and uses an inverted triangle load pattern such that α = 2/3, as in (20), using η = 2.0 and ε_y = 0.2% leads to:

$$\frac{\delta_e}{h_w} = \frac{28}{270}\mathrm{A}_r(\%) \approx 0.1\,\mathrm{A}_r(\%) \qquad (26)$$

Again, for flanged walls with the flange in compression, the right-hand side of (26) should be multiplied by 3/4. By comparison of (25) with (26), it is found that when neglecting the dynamic effects, δ_e is overestimated by approximately 70%, under all the assumptions considered in the simplified versions of (20).

Limitations of the approach and further research

The assumptions made for constructing the formulae introduced in this paper are discussed to open opportunities for its evaluation and rational criticism in the sense of Popper (1963) (see also Miller (1994) and Verdugo (1995)).

Firstly, it was assumed that the cross-section stiffness EI is constant along the height of the wall: this is not true when the wall is placed within a building, in particular, as the axial load decreases with the height, and normally so does the amount of longitudinal reinforcing steel. This results in a decreased M_n in the upper floors, and consequently a reduced flexural stiffness EI, due to both effects. The implication is that, for the same externally imposed lateral load pattern, the curvature of the wall will be larger along its height when the strength variation is included compared to when is not. However, this would traduce into greater values of δ_{te} compared to those calculated with the aforementioned assumption. Hence, the approximation leads to conservative results.

Secondly, the value of ω_v considered for developing the simplified formulas, might not be appropriate for all cases, and should be understood as a 'current' upper bound, which could well be increased in the future, depending on the evidence.

Lastly, any connection of the wall with the surrounding structure is neglected. Therefore, at least the coupling effects of the floor slabs and/or beams, which can be more pronounced for walls ending in the façade of a building, are neglected. This, in turn, means neglecting the variation of the axial load imposed to the wall by coupling with the rest of the structure via these members.

As future research to cover some of the aspects outlined above, and critically evaluating the proposed formulae, numerical simulations of a building with rectangular walls of different aspect ratios A_r, modelled with macro and fibre elements, are currently under preparation. Additionally, collaborative efforts with researchers working on the same topic, are expected to provide a more comprehensive evaluation of the approach, when including the results of nonlinear dynamic analyses of buildings with asymmetric (flanged) walls.

Summary

This article provides a simple formulation for calculating the maximum elastic (yielding) roof displacement of a slender cantilever RC wall, accounting for dynamic effects. This displacement is calculated as a function of the equivalent lateral force resulting from a certain lateral load distribution, and its equivalent height, measured from the critical section of the wall. The equivalent height is firstly calculated for a load distribution associated to the first mode of vibration (*e.g.* an inversed triangle), and is subsequently reduced to account for a load pattern representative of a higher-mode response. The ratio between both heights corresponds to the dynamic amplification factor (ω_v > 1.0) used in capacity-based design for shear actions, as proposed in the past by other researchers. An expression for the maximum lateral roof elastic drift ratio of a slender cantilever wall is formulated, including dimensionless numbers only, using a well-established expression for the yielding curvature of RC members, and assuming that the nominal yielding point at the base of the wall is reached when the equivalent load acts at the reduced height. Developing a common parameter for comparison, and using a simple numerical example, it is shown that the proposed novel formula predicts smaller maximum elastic deflections compared to expressions previously presented in the literature. Finally, different versions of the proposed formulae, with various levels of simplification, are presented, aiming at its consideration for its use within the Chilean RC code, after a thorough critical evaluation with nonlinear analyses.

Acknowledgements

The author would like to thank Professor Patricio Bonelli and Ms. Rocío Alvarez for presenting him the problem addressed in this article. Thanks to Mr. Joaquín Valenzuela for a suggestion that helped improving Figure 1.

References

DS61 (2011). Decreto Supremo 61. Reglamento que fija el diseño sísmico de edificios y deroga decreto nº 117, de 2010. Ministerio de Vivienda y Urbanismo, Diario Oficial de la República de Chile, Nº 40.133, 8-12 (in Spanish)

INN (2009). Seismic design of buildings. Norma Chilena Oficial NCh 433 Of.96 modificada 2009, Instituto Nacional de Normalización, Santiago, Chile (in Spanish)

Miller, D.W. (1994). *Critical rationalism: a restatement and defence*. Open Court Publishing, Peru, Illinois, USA

Massone, L.M., Alfaro, J.I., Ahumada, M. and Rojas, F. (2015). Estimación del desplazamiento lateral y curvatura en muros esbeltos. XI *Congreso Chileno de Sismología e Ingeniería Sísmica* ACHISINA 2015, Santiago, paper N°29

Park, R., and Paulay, T. (1975). *Reinforced concrete structures*. John Wiley & Sons, New York

Paulay, T. and Priestley, M.J.N. (1992). *Seismic design of reinforced concrete and masonry buildings*. John Wiley & Sons, New York

Paulay, T. and Priestley, M.J.N. (1993). Stability of slender reinforced concrete walls. *ACI Structural Journal* **90**(4), 385-392

Paulay, T. (2002). A displacement-focused seismic design of mixed building systems. *Earthquake Spectra* **18**(4), 689-718

Popper, K. (1963). *Conjectures and refutations.* Routledge, New York

Priestley, M.J.N. and Kowalsky, M.J. (1998). Aspects of drift and ductility capacity of rectangular cantilever structural walls. *Bulletin of the New Zealand Society for Earthquake Engineering* **31**(2), 73-85

Priestley, M.J.N. (2003). Myths and fallacies in earthquake engineering, revisited. The Mallet Milne lecture 2003, IUSS Press, Pavia, Italy

Priestley, M.J.N., Calvi, J.M. and Kowalsky, M.J. (2007). *Displacement-based seismic design of structures*. IUSS Press, Pavia, Italy

Quintana Gallo, P. (2008). *Analytical evaluation of damage in a reinforced concrete building*. MSc thesis, Universidad Técnica Federico Santa María, Valparaíso, Chile (in Spanish)

Quintana Gallo, P. (2014). *The nonlinear dynamics involved in the seismic assessment and retrofit of reinforced concrete buildings*. PhD thesis, University of Canterbury, Christchurch, New Zealand

Rutenberg, A. (2013). Seismic shear forces on RC walls: review and bibliography. *Bulletin of Earthquake Engineering* **11**(5), 1727-1751

SNZ 3101 (2006). Concrete Structures Standard. Standards New Zealand, Wellington, New Zealand

Verdugo, C. (1996). La Filosofía de la Ciencia de Popper. *Estudios Públicos* **62**, 181-195 (in Spanish)

Wallace, J.W. and Moehle, J.P. (1992). Ductility and detailing requirements of bearing wall buildings. *Journal of Structural Engineering* **118**(6), 1625-1644