A NEW RELATIVISTIC FIELD THEORY OF THE ELECTRON

UNA NUEVA TEORÍA RELATIVÍSTICA DE CAMPO PARA EL ELECTRÓN

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RESUMEN

En este trabajo, se presenta un examen cualitativo sobre una nueva Teoría General Relativística para el electrón con la obtención de la ecuación de Dirac a partir de los campos electromagnéticos con el campo eléctrico paralelo al campo magnético. El principio rector es el de la relatividad general, y la principal hipótesis es que de las ecuaciones fundamentales se desprende la teoría de Dirac y la teoría de Maxwell - Lorentz como de dos casos especiales. cuidadando la coherencia y compatibilidad entre las condiciones en las que las ecuaciones fundamentales se reducen a la ecuación de Dirac y las ecuaciones de Maxwell - Lorentz. Se espera que la presente investigación arroje alguna luz sobre los desconcertante dificultades a las que nos encontramos en la comprensión de el comportamiento de un electrón exclusivamente en función de la ecuación de Dirac y las ecuaciones de Maxwell - Lorentz. Más allá de esto, se tiene como objetivo el investigar la posibilidad de que otras partículas elementales se puedan regir por las mismas ecuaciones fundamentales bajo variadas condiciones restrictivas.

Palabras clave: Ecuación de Dirac, Tensor de materia, sistema Einstein-Maxwell.

ABSTRACT

In this paper, we present a qualitative discussion on a new General Relativistic Field Theory for the electron, obtaining the Dirac equation from electromagnetic fields with the electric field parallel to the magnetic field. The guiding principle is that of general relativity, and the main hypothesis is that the fundamental equations embrace the Dirac theory and the Maxwell-Lorentz theory as of two special cases respectively. We concern ourselves with the consistency and compatibility among those conditions under which the fundamental equations are reduced to the Dirac equation and the Maxwell-Lorentz equations. We expect that the present investigation will shed some light on those perplexing difficulties which we encounter in comprehending the behavior of an electron solely according to the Dirac equation and the Maxwell-Lorentz equations. Beyond this, we have a goal to investigate the possibility that other elementary particles are governed by the same fundamental equations under varied restrictive conditions.

Keywords: Dirac equation, matter tensor, Einstein-Maxwell system.

INTRODUCTION

Einstein's Dream

Albert Einstein spent several years of his life trying to develop a theory which would relate electromagnetism and gravity to a common "unified field". Hence the name ***unified field theory***.

After Einstein finished his first article on the unified field theory in 1922, despite criticism he spent much of the second half of his life pursuing the development of the unified field theory besides the discussion of completeness of quantum mechanics. In the first several years, he was very optimistic, thought success would come soon, but he found it was full of difficulties afterwards. He considered mathematical tools in being was not sufficient, then turned to study mathematics, but never obtained any result with real physical sense. Because Einstein wanted to found an encompassing mathematical construct that would unite not only gravitational field but also electromagnetic field under a single set of equations, but then the task has become even more difficult, with the discovery of two other basic field: the weak interaction field and strong interaction field. Most physicists thought Einstein's

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quest was hopeless, and in fact he never succeeded. But Einstein was convinced such a basic harmony and simplicity existed in nature, he kept his chin up, went ahead along his own road. Because he was apart from the mainstream of physical research - quantum field theory, he was very alone in his old age, but he was fearless. He still prepared to keep on his mathematical calculation of unified field theory on his sickbed until the day before his death. He said with a sigh before his death: I cannot finish this work, it will be forgotten, but it will be rediscovered in the future. Einstein did manage to develop a theory which "wrapped" electromagnetism and gravitation into a common metric tensor. In one of his formulations of a unified field theory (called Einstein-Schrodinger Theory), gravitation was wrapped into the symmetric part of the metric tensor, while electromagnetism was wrapped into the antisymmetric part of the metric tensor. This wrapping is possible because electromagnetism and gravity share some mathematical similarities. They both have a stress-energy tensor. The electric charge is analogous to the gravitational mass. The magnetic moment is analogous to the angular momentum moment. The electric potential and electric field are analogous to the gravitational potential and gravitational field, respectively. Finally, the magnetic field is analogous to the magneto-gravitic field.

The mathematical wrapper which Einstein developed exploits this analogy. However, the analogy between electromagnetism and gravity breaks down at higher field strengths when nonlinear field effects set in. As a result, Einstein-Schrödinger theory correctly describes electromagnetism and gravity at low field strengths where they are not coupled to each other. However, it does not describe the interactions between electromagnetism and gravitation which occur at higher field strengths. Thus, Einstein-Schrödinger theory achieved an approximate mathematical unification, but no real physical unification of electromagnetism and gravity. In this sense, it did not really achieve its objective.

Kaluza and Klein developed an alternative wrapper for electromagnetism and gravitation. Instead of wrapping electromagnetism into the antisymmetric part of the metric tensor, they retained a symmetric metric tensor but added a fifth dimension. They were able to show that Maxwell's Laws and General Relativity can be expressed in terms of their five-dimensional metric tensor. Again, this exploits the analogies between electromagnetism and gravity.

The problem with Einstein's unified field theory and Kaluza-Klein's unified field theory is that they don't address the fundamental issue. They still treat gravitation and electromagnetism as two completely separate interactions. Neither theory can tell you how a gravitational field is fundamentally produced by a charged particle (electron).

Today, the search for a unified field theory has been replaced by loftier goals. Physicists are now looking for a so-called Theory of Everything (TOE) which will unify not only electromagnetism and gravity, but also the nuclear interactions and other potential physical forces such as inflation and "dark energy". At the time of Einstein, modern particle physics had not yet been developed and the strong and weak nuclear interactions were not well understood. Within of the unified program a fundamental question was if gravitational fields did play an essential part in the structure of the elementary particles of matter (electron). The first unimodular theory was developed by Einstein in 1919, assuming as source the Maxwell tensor, where the quantum electron theory was not reproduced. Thus, as stated by Einstein in 1919, "we cannot arrive at a theory of the electron [and matter generally] by restricting ourselves to the electromagnetic components of the Maxwell-Lorentz theory, as has long been known" [6].

Motivation

In the beginning of this century, Lorentz, Poincaré, Abraham, Mie and others attempted to show that the constitution of an electron be explained as a field of electromagnetic nature. In order to make the motion of the electron special-relativistic, however, it was necessary to consider a mechanical core (Poincaré) or to introduce some nonlinearity (Mie) in the electromagnetic field under consideration [1-3]. To overcome these difficulties appeared to have completely been resolved with the Dirac equation for the electron discovered in 1928. It has conventionally been believed that the information of an electron near its core is fully provided by the Dirac equation. The notion of the electron formed by the conventional interpretation of the Dirac equation is hardly acceptable as rational and feasible. Instead, in an accompanying paper, it was shown that the Dirac equation has a solution that indicates that an electron is a field localized in space and deformable and that the motion of this "elementary field" is determinate and causal. Moreover, is shown, it is possible to regard the field governed by the Dirac equation as if electromagnetic. This possibility is not surprising if we note that the intrinsic magnetic moment of an electron is a notion to be comprehended only in the context of Faraday-Maxwell-Lorentz's theory of electricity and magnetism. Thus we are led to infer the following: An electron is a localized field of which some part remote from its center may well be regarded as normal electromagnetic, and some other part near its center is governed by the Dirac equation derived from parallel fields. The connection between the two parts must be continuous and gradual, and there is no clear-cut border between them. A real electron, as a whole, must be a unified field governed by a common set of partial differential equations. It is important to anticipate the possibility that those fundamental equations governing the field be reduced to the Maxwell-Lorentz equations under a restrictive condition and to the Dirac equation under another restrictive condition. Although with the early theories of Einstein and others [6-7], there is no deductive way of giving the fundamental equations, it is not difficult to anticipate the following:

a. The electronic mass has its representation in the Dirac equation, but not in the Maxwell-Lorentz equations. On the other hand, the electronic charge is seen in the Maxwell-Lorentz equations, but not in the Dirac equation for a free electron. We infer from these observations that the electronic mass and charge are approximate substitutes of field variables that are functions of time and space in the fundamental equations. Only because the variables are comparatively less variants, they may be replaced with constants as depending on conditions of observation.

b. The Maxwell-Lorentz equations are covariant under the Lorentz transformation, however, the covariance of the Dirac equation under the same transformation is conditional. Besides, the field variables in Maxwell-Lorentz equations and those in the Dirac equation are apparently of different characteristics under the Lorentz transformation. In order to embrace those two sets of equations as of special cases, the fundamental equations must be formed in a geometrical frame less restrictive than the Euclidean, i.e., as covariant for observers in varied conditions [8-9].

These difficulties are overcomes with our Maxwell's Equations with parallel electromagnetic fields, (see accompanying paper). Considering those observations in the above, it is significant to recall the demonstration of the similarity between the Dirac field and the electromagnetic field. In the demonstration, we see a clue to electing a set of fundamental equations that are reducible to both the Dirac equation and the Maxwell-Lorentz equations. In paper (Physical interpretation of the Dirac equation with electromagnetic mass), we considered the Dirac equation for a free electron derived from Maxwell's equations when the electric field is parallel to the magnetic field.

The Nature of the Investigation

If one accepts as valid the principle of relativity, i.e., the principle of covariance of the laws under coordinate transformations, the choice of a proper scheme of geometry is an essential part of the task of constructing the fundamental equations concerned. In this respect, it is significant to recall that the Dirac equation is not completely covariant under the Lorentz transformation. It appears that the range of the meaning implied by the Dirac equation can no longer be confined in the Euclid space. This situation suggests first that the scheme of geometry be properly generalized and then that the Dirac equation be modified accordingly. We expect, in this way, that the fundamental equations thus found will be able to embrace the Dirac equation and the Maxwell-Lorentz equations as of two special cases respectively.

In a geometrical scheme more general than the Euclidean, each component of the metric tensor g_{ij} is a function of space-time coordinates. Therefore, it seems to be sensible to expect that any matter field, with no exception, is accompanied by a gravity field. The fundamental equations govern simultaneously the matter field and the metric field is the equation $R_{ij} - \frac{1}{2} g_{ij} R = -kT_{ij}$ proposed earlier by Einstein [10-11]. The left hand side of the equation is sometimes called the Einstein tensor G_{ij}. One might surmise that a matter field determines uniquely the Einstein tensor of the space where the matter field is located. Thus it appears that the Einstein tensor can be the representation or the image of the matter field. But the uniqueness of the relation between a matter field and the resulting Einstein tensor is unknown. We note that the equations to be found must be regarded only as of an approximate means of representing the reality concerned. (None of the equations utilized in physics may escape this fate.) Therefore, even when a field, e.g., of an electron, governed by those equations has a singularity that implies a strong distortion of space curvature, one can not immediately conclude that the real field, expected to be represented by the solution, has the same singularity.

The field equations of general relativity are rarely used without simplifying assumptions. The most common application treats of a mass, sufficiently distant from other masses, so as to move uniformly in a straight line. All applications of special relativity are of this type, in order to stay in Minkowski space-time. A body that moves inertially (or at rest) is thus assumed to have four-dimensionally straight world lines from which they deviate only under acceleration or rotation. The well-known Minkowski diagram of special relativity is a graphical representation of this assumption and therefore refers to a highly idealized situation, only realized in isolated free fall or improbable regions of deep intergalactic space.

In the real world the stress tensor never vanishes and so requires a non- vanishing curvature tensor under all circumstances. Alternatively, the concept of mass is strictly undefined in Minkowski space-time. Any mass point in Minkowski space disperses spontaneously, which means that it has a space-like rather than a time-like world line. In perfect analogy a mass point can be viewed as a local distortion of space-time. In euclidean space it can be smoothed away without leaving any trace, but not on a curved manifold. Mass generation therefore resembles distortion of a euclidean cover when spread across a non-euclidean surface. A given degree of curvature then corresponds to creation of a constant quantity of matter, or a constant measure of misfit between cover and surface, that cannot be smoothed away.

Here, a strain field appears in the curved surface. At any point on the curved manifold the gradient of the strain field is perpendicular to the tangent vector and coincides with the axis of the local light cone. To relieve the stress, the natural response of the mass point is displacement along the stress gradient and hence it traces out a time-like world line at constant spatial coordinates. This displacement, along the time coordinate only, is the arrow of time, which appears as a direct consequence of the curvature of space. There is no time in euclidean space.

The primary cause of mass generation by space curvature is elimination of the strict orthogonality between time and space coordinates which allows the strain field (mass point) to acquire complementary time-like and space-like attributes. This is the mechanism envisaged by Corben [4] as a model for creating mass through relativistically invariant self-trapping of a free bradyon and a free tachyon, (time-like and space-like waves).

The essence of the argument advanced here is that real world-space is not euclidean and that space is generally curved into the time dimension, consistent with the theory of general relativity. The curvature may not be sufficient to become obvious in a local context. However, it is sufficient to break the time-reversal symmetry that seems to characterize the laws of physics. Not only does it cause perpetual time with respect to all mass, but actually identifies a fixed direction for this It creates an arrow of time and thereby eliminates an inconsistency in the logic of physics: how reversible microscopic laws can underpin an irreversible macroscopic world. General curvature of space breaks the time-reversal symmetry and produces chiral space, manifest in the right-hand force rule of electromagnetism. The fact that most other fundamental laws of physics do not refer the **chirality** of space, nor the arrow of time, confirms that the curvature on a local scale is barely detectable.

The one exception to apparent time-reversal symmetry is the law of entropy.

It has been stated [1] that "... the second law of thermodynamics is excluded from the classication fundamental due to its statistical nature". This is an unconvincing explanation and the curved-space argument provides a better mechanism for entropy production. In any curved-space manifold gradient vectors drive time-like displacement of separate particles along non-parallel world lines. Even among pairs of stationary particles three-dimensional line elements therefore do not remain invariant over a period of time. An initially stationary array of non-interacting particles (ideal gas) spontaneously generates relative internal (zero point) motion leading to chaotic distribution in a container, or spontaneous dispersal in the open. Where local interactions constrain dispersal, zero-point vibration develops. This intrinsic microscopic instability, caused by the curvature of space, is the source of entropy.

The conclusions reached here are clearly related to those of Prigogine [5] who deduced that the irreversible creation of matter generates cosmological entropy and that the arrow of time is provided by the transformation of gravitational energy into matter. The difference is that Progonine's result was obtained by incorporating the second law of thermodynamics into the relativistic field equations, whereas the present model makes no assumption about macroscopic behaviour.

Theses observations, usual in classical mechanics, are significant in evaluating Einstein's attempt recollected in the following.

Recollection of Einstein's Attempt

It seems that Einstein devoted the last twenty years, at least, of his life to the attempt of materializing his deterministic view of particles. Einstein explicitly used the term "unified field theory" (gravitation-electromagnetism) in the title of a publication for the first time in 1925. Ten more papers appeared in which the term is used in the title, but Einstein had dealt with the topic already in half a dozen publications before 1925 . In total he wrote more than forty technical papers on the subject. This work represents roughly a fourth of his overall oeuvre of original research articles, and about half of his scientific production published after 1920. As is well known, however, the endeavor was not fruitful. In retrospect, the cause of his difficulty appears to be in his interpretation of Schrödinger's wave equation. A clue to knowing Einstein's interpretation in question is found in an essay published by him in 1936 (Einstein, 1936). His interpretation of wave mechanics may be summarized as follows:

i) The wave function does not in any way describe the condition of a single system; it relates rather to many systems, an ensemble of systems, in the same sense as of statistical mechanics, so Schrödinger's equation determines the time variation that is experienced by an ensemble of systems.

ii) Quantum mechanics will not be the point of departure in the search of the foundation of quantum-mechanical phenomena, just as one cannot go from thermodynamics to the foundation of mechanics, so there must be a field theory that results in a way of representing particles and the representation must be free of singularities. The foundation of the theory is given by the differential equations of the field, and the theory leads also to quantum mechanics in the same way as classical mechanics of particles leads to thermodynamics.

Einstein emphasized often that the field in question must be free of singularities. His reasoning seems to be based on the following two observations: Conventional wave functions in quantum mechanics are free of singularities. On the other hand, in his general theory of relativity completed in 1916, the differential equations of the metric space completely replace the Newton theory of the motion of celestial bodies, if the masses are substituted with singularities of the field; those equations contain the law of force as well as the law of motion while eliminating inertial systems. His theory with $T_{ij} = T_{ij}^{\max well}$, however, does not explain quantum-mechanical phenomena, and is not satisfactory (unimodular theory). Considering these two facts, Einstein had a conjecture that a satisfactory theory be obtained by modifying the general theory of relativity so that the singularities do not arise in a field determined by the differential equations of the metric space. He assumed that the desirable modification be made by eliminating the symmetry condition of the metric tensor from the general theory of relativity completed in 1916. According to Einstein, equations of such complexity as expected can be found only through the discovery of a logically simple mathematical scheme that determines the equations of physics completely or almost completely. Once one has a proper mathematical scheme, one requires only little knowledge of physical facts for setting up a proper theory.

In 1948, near the end of his life, Einstein thought that he had success in formulating a satisfactory scheme of geometry in which the metric tensor is no longer symmetric. He hoped that this geometry could provide the framework in which the new theory of physics be established. Unfortunately, however, the result was disappointing; a stationary field free from singularities could never represent a mass different from zero. We thus recognize that Einstein's view of conventional quantum mechanics is partially right, and a causal and determinative law is underlying conventional quantum-mechanical phenomena of the electron. Considering this, it appears to be a serious misjudgment of Einstein to attribute immediately the cause of singularities to the symmetry condition of the metric tensor in the Riemann geometry.

Now, we can say that the general solution of a partial differential equation contains a set of functions whose forms are not determined by the equation but by initial and boundary conditions. A physically significant solution is a particular solution that satisfies proper initial and boundary conditions. It is a significant event in the history of physics that Einstein had persistently failed to recognize the significance of initial and boundary conditions in interpreting physical laws. We see the same failure in Dirac's interpretation of the Dirac equation for the electron if we not considerer that the Dirac equation is derived from chiral electromagnetic fields with $\boldsymbol{E} \square \boldsymbol{B}$.

FUNDAMENTAL EQUATIONS

In the following investigation, the variables are in general defined as tensors in a four-dimensional Riemannian space. The mathematical treatment of them follows the ordinary rule of tensor calculus. For the convenience of reference, the mathematical symbols employed are mostly similar to those in (Møller, [9-11]), unless otherwise specified.

Those equations are mutually coupled, and the strong tendency of the electron to be a localized and stable field must be effected by the characteristics of those equations and proper boundary and initial conditions.

For formulating the fundamental equations, it is customary to rely on Hamilton's principle of variation of deriving covariant equations from a Lagrangian function. But the choice of the Lagrangian function is arbitrary, and so is of variation methods. There is no assurance of uniqueness of the result. As Eddington remarked earlier [12], the physical significance of the method is unknown and doubtful, particularly when we have no means of evaluating those resulting equations immediately and directly in comparison with empirical information. Our experience in this field of physics is yet naive; instead of taking any axiomatic approach, it seems to be desirable to continue an effort of reflecting on the physical reality via equations known thus far. The guiding principle is that of general relativity, and the main hypothesis is that the fundamental equations embrace the Dirac equation and the Maxwell-Lorentz equations as of two special cases respectively. Although we do not intend to compare solutions of the fundamental equations directly with empirical information, we concern ourselves with the consistency and compatibility among those conditions under which the fundamental equations are reduced to the Dirac equation and the Maxwell-Lorentz equations. We expect that the present investigation will shed some light on those perplexing difficulties which we encounter in comprehending the behavior of an electron solely according to the Dirac equation and the Maxwell-Lorentz equations. Beyond this, we have an ambition to investigate the possibility that other elementary particles are governed by the same fundamental equations under varied restrictive conditions.

The Matter Field

The equations for the matter field and those for the metric tensor field are intimately coupled together. In a conventional sense, however, we may call the following the equations for the matter field:

$$\frac{1}{\sqrt{-g}}\frac{\partial\left(\sqrt{-g}F^{ij}\right)}{\partial x^j}-g^{ij}\frac{\partial\eta}{\partial x^j}=0\,, \tag{1}$$

$$\frac{1}{\sqrt{-g}}\frac{\partial\left(\sqrt{-g}F^{*ij}\right)}{\partial x^j}-g^{ij}\frac{\partial\xi}{\partial x^j}=0\,. \tag{2}$$

In these equations, g is the determinant of the metric tensor g^{ij}; F^{ij} is an antisymmetric tensor and F^{*ij} is conjugate to F^{ij}; ξ and η are scalars. One might ask why these equations are fundamental. The answer is simple: Firstly, these equations are covariant in the Riemannian sense; secondly, by considering the current s^i_{elect} for $g^{ij}\partial\eta/\partial x^j$ and by assuming ξ^i_{magnet} for $g^{ij}\dfrac{\partial\xi}{\partial x^j}$, these equations can be as the Riemannian generalization of the Maxwell-Lorentz equations. However, we do not immediately relate these equations to the Maxwell-Lorentz equations; a physical consideration is needed prior to doing so.

We expect that those equations in the above will eventually be reduced to the Dirac equation and also to the Maxwell-Lorentz equations, and write for F^{ij}

$$F^{ij}=\begin{pmatrix}0 & Q_z & -Q_y & -P_x\\ -Q_z & 0 & Q_x & -P_y\\ Q_y & -Q_x & 0 & -P_z\\ P_x & P_y & P_z & 0\end{pmatrix}. \tag{3}$$

Considering

$$\begin{aligned}F^{*ij}&=g^{ik}g^{jm}F^*_{km}\\ &=\frac{1}{2}\sqrt{-g}\,g^{ik}g^{jm}\delta_{kmst}F^{st}\end{aligned} \tag{4}$$

where δ_{kmst} is the Levi-Civita symbol, we have

$$F^*_{ij}=\begin{pmatrix}0 & -P_z & P_y & Q_x\\ P_z & 0 & -P_x & Q_y\\ -P_y & P_x & 0 & Q_z\\ -Q_x & -Q_y & -Q_z & 0\end{pmatrix}\times\sqrt{-g}\,,$$

$$F^{*ij}=\begin{pmatrix}0 & -P'_z & P'_y & -Q'_x\\ P'_z & 0 & -P'_x & -Q'_y\\ -P'_y & P'_x & 0 & -Q'_z\\ Q'_x & Q'_y & Q'_z & 0\end{pmatrix}. \tag{5}$$

From here on, we shall often write $\vec{P}$ for (P_x, P_y, P_z) and $\vec{Q}$ for (Q_x, Q_y, Q_z) simply for the sake of convenience, although they are not three-vectors. We note that in general.

$$\vec{P}\neq\vec{P}'\,,\qquad \vec{Q}\neq\vec{Q}' \tag{6}$$

However, if

$$\vec{\mathrm{P}}=\vec{\mathrm{P}}'=\mathbf{E},\qquad \vec{Q}=\vec{Q}'=\mathbf{B} \tag{7}$$

We have a nice approximation. We expect the equivalence between the two sets of equations will be established, if the metric tensor field be properly evaluated in the following.

The Metric Tensor Field

As is well known, Einstein in 1916 proposed an equation for the metric tensor [6].

$$R_{ij}-\frac{1}{2}g_{ij}R=-kT_{ij} \tag{8}$$

where R_{ij} is the contracted curvature tensor, R is the curvature scalar, and T_{ij} is the energy-momentum tensor of the matter field. Einstein gave this equation by considering that the only fundamental tensors that do not contain derivatives of g_{ij} beyond the second order are functions of g_{ij} and the Riemann-Christoffel curvature tensor and that the equation is analogous to the Poisson equation for the gravitational field to the non-relativistic limit. It seems that Einstein proposed this equation for the purpose of solving cosmological problems, i.e., the structure of the universe as a whole [6]. Therefore, T_{ij} is expected to be a known tensor supplied from the data of astronomical observation of the average mass distribution. Schwarzschild showed that the equation with $T_{ij}=0$ has a particular solution that expresses properly the gravity field induced by a material point [12].

Only when Eq. (8) is considered simultaneously with Eqs. (1) and (2), the equation for an elementary particle may be solved. If it is noticed that Eq. (8) alone consists of ten simultaneous partial differential equations of the second order, the analytical treatment of those equations concerned is an extremely difficult task. Moreover, it was not completely known how T_{ij} is to be constructed in terms of F^{ij}, η and ξ in the Einstein epoch. (As noted a short time ago, the variation method is not a decisive one.) Our present purpose is to show that the Dirac equation and the Maxwell-Lorentz equations, which are covariant only in the Euclidean sense, are both attainable by linearization of the same one set of nonlinear equations covariant in a non-Euclidean sense. From this viewpoint, we consider that it may not be necessary that the fundamental equations are immediately covariant in the Riemannian sense. There may be schemes of geometry that are more general than the Euclidean and less than the Riemannian. It is noted that, because of the restrictive conditions, viz., Eq. (8), Einstein's geometry is less general than the Riemannian [12]. According to Einstein, the Einstein tensor, the left hand side of Eq. (8), should vanish in a space empty of matter. On the other hand, in the Riemann geometry, it does not vanish in general. (In the Riemann geometry, the idea of matter does not exist.) However, the covariant divergence of the Einstein tensor vanishes always in the Riemann geometry as well as in Einstein's [10]. As noted earlier, Einstein chose Eq. (8) as one of the possibly simplest equations. Our conjecture is even when we adopt Einstein's equation the obtained equation is adequate completely for describing the field extremely near the center of the core of an electron. Instead of taking an axiomatic approach, it is essential to study carefully the circumstances under which the present investigation is motivated.

Later Einstein (1919) attempted to investigate the structure of an elementary particle as based on the same equation. There, however, he did not pay much attention to T_{ij}. He simply speculated that the matter field is an electromagnetic field, using a unimodular theory with $T_{ij}=T_{ij}^{\max well}$ and the magnetic field $\boldsymbol{B}$ perpendicular to electric field $\boldsymbol{E}$, ($\boldsymbol{E} \perp \boldsymbol{B}$). Contrary to Einstein conjecture, in our present problem in which an electron is considered to be a small universe, we considerer $\boldsymbol{E} \parallel \boldsymbol{B}$, ie, we suppose that the electron –positron equation is the Dirac equation if only if it is derived from electromagnetic fields with $\boldsymbol{E} \parallel \boldsymbol{B}$, inserted in the original Einstein equation $R_{ij}-\frac{1}{2}g_{ij}R=-kT_{ij}$, with $T_{ij}=T_{ij}^{Maxwell}\big|_{\boldsymbol{E}=i\boldsymbol{B}}$.That means $F^{ij}=iF^{*ij}$, where $i=\sqrt{-1}$, and s^{i}_{elect}, ξ^{i}_{magnet}, given by

$$\frac{\partial F^{\mu\nu}}{\partial x^{\nu}}=\frac{4\pi}{c}J^{\mu}_{e}=-\frac{imc}{\hbar}E^{\mu}_{e}=s^{i}_{elect} \tag{9}$$

$$\frac{\partial \tilde{F}^{\mu\nu}}{\partial x^{\nu}}=\frac{4\pi}{c}J^{\mu}_{m}=\frac{imc}{\hbar}B^{\mu}_{m}=\xi^{i}_{magnet} \tag{10}$$

(For the specific demonstration see the article Maxwell's theory with chiral current)

Thus, contrary to Einstein equation for the electron (unimodular theory) [13], i.e., the equations for the matter field and those for the metric tensor, do not contain Planck's constant *h*, the electronic mass *m* and charge *e*, our equation (8), (9) and (10) contain *h, m, e, which are essential to obtain the Dirac equation.*

With equations (9-10), it's possible to show that an electron is like a toroid with $\boldsymbol{E} \parallel \boldsymbol{B}$, spin ½, without radiation and $r_p=T=\hbar/2mc$ (figure 1).

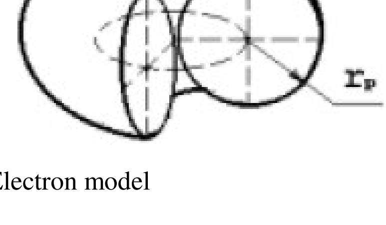

Figure1. Electron model

Assumptions as regards the metric tensor field

According to the above results and considerations, we assume that the field in question is spatially localized. In the space outside the field, the Riemann-Christoffel tensor is negligibly small. It would be more reasonable to regard a part of the space as outside when the Riemann-Christoffel tensor is negligibly small in that part. Also we note that, owing to the other bodies of matter contained in the universe, the tensor in question does not completely vanish at any point of the space. But our interest is in the local field, the electron. Hence, we ignore the curvature of the global scale, and may consider an inertial frame of reference outside the electron. (Einstein, perhaps due to his esteem of Ernst Mach, did not necessarily seem to think that the field of an electron can be completely closed and sustained by itself [12, 13]). If we consider an electron fixed to an inertial frame of reference, the electron appears to be free from the influence of the external universe. Classically, if we consider the internal structure of the electron, the situation is not necessarily so simple. It seems possible that a portion of the electron is in acceleration relative to the inertial frame reference in the same way as a portion of a spinning top resting as a whole on the inertial frame is. Such a classical-mechanical structure is inconceivable. However, it is sensible then to assume that the electron has a stable structure with its own permanent gravity field, as independent of the influence of the external universe.

CONCLUSION

Thus we are presented a new theory called “Teoría Total Simplificada” (TTS) based on chiral electrodynamic which reproduces at the first time the Dirac equation for the electron unifying the gravity with electromagnetism [14-16].

REFERENCES

[1] W.G. Dixon, Special Relativity. University Press, Cambridge. 1978.

[2] R. Adler, M. Bazin and M. Schiffer, Introduction to General Relativity. McGraw-Hill, NY. 1965.

[3] M. Friedman. Foundations of Space-time theories. Princeton U.P., Princeton, NJ. 1983.

[4] E. Witten. “Duality, Spacetime and Quantum Mechanics”. Physics Today. Vol. 50, pp. 28-33. 1997.

[5] S. Weinberg. A Unified Physics by 2050”. Sci. Am. Vol. 281 Nº 6, pp. 68-75. December, 1999.

[6] A. Einstein. “Do Gravitational Fields Play an Essential Part in The Structure of the Elementary Particles of Matter?” The Principle of Relativity. Dover, pp. 191-198. 1952.

[7] P.G. Bergmann and R. Thomson. Spin and Angular Momentum in General Relativity. Phys. Rev. Vol. 2 Nº 89, pp. 400-407. 1953.

[8] J. N. Goldberg, Conservation Laws in General Relativity, Phys. Rev. Vol. 2 Nº 111, pp. 315-320. 1958.

[9] Toyoki Koga. “A relativistic field theory of the electron”. Intl. J. of Theo. Physics. Vol. 15, pp. 99-119. 1976.

[10] C. Møller. “Further Remarks on the Localization of the Energy in the General Theory of Relativity”. Ann. Physics. Vol. 12, pp. 118-133. 1961.

[11] C. Møller. Conservation Laws and Absolute Parallelism in General Relativity. Mat. Fys. Skr. Danske. Vid. Selsk. Vol. 1 Nº 10, pp. 1-50. 1961.

[12] A.S. Eddington. The Mathematical Theory of Relativity. Cambridge University Press, Cambridge. 1921.

[13] A. Einstein. “DieKompatibilität der Feldgleichungen in der einheitlichen Feldtheorie”. Preuss. Akad. Wiss. Berlin, Phys. Math. Klasse, Sitzber, pp. 18-23. 1930.

[14] H. Torres-Silva. “Electrodinámica quiral: eslabón para la unificación del electromagnetismo y la gravitación”. Ingeniare. Rev. chil. ing. Vol. 16 Nº 1, pp. 6-23. 2008.

[15] H. Torres-Silva. “The close relation between the maxwell system and the dirac equation when $\vec{E}$ is parallel to $\vec{H}$ ”. Ingeniare. Rev. chil. ing. Vol. 16 Nº 1, pp. 43-47. 2008.

[16] H. Torres-Silva. “Chiral field ideas toward a theory of MATTER”. Ingeniare. Rev. chil. ing. Vol. 16 Nº 1, pp. 36-42. 2008.