SHAPIRO EFFECT IN INDUCTIVE QUANTUM CIRCUITS WITH CHARGE DISCRETENESS

EFECTO SHAPIRO EN CIRCUITOS CUÁNTICOS INDUCTIVOS CON CARGA DISCRETA

Kristopher Chandía Valenzuela[1] **Juan César Flores Araya**[2] **Edmundo Lazo Núñez**[2]

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RESUMEN

Es sabido que circuitos cuánticos inductivos con carga discreta, cuando se someten a un voltaje continuo externo, presentan oscilaciones de Bloch en la corriente. En este trabajo se considera, además, la superposición de un voltaje alterno en el circuito. El efecto Shapiro, relacionado con la existencia de resonancias, es encontrado de modo explícito. Sorprendentemente, en el límite de bajas frecuencias (sin resonancia) la corriente eléctrica promediada existe y tiene siempre el mismo signo. Eventualmente, esto entrega un método experimental para medir los efectos de la discretización de la carga en circuitos cuánticos mesoscópicos.

Palabras clave: Circuitos mesoscópicos, teoría de circuitos cuánticos, efecto Shapiro, carga discreta.

ABSTRACT

As it is known, quantum inductive circuits with charge discreteness show Bloch-like oscillations in electrical current under a dc external voltage. In this paper, the effect of a superimposed ac voltage in the circuit is considered. The Shapiro effect is found to be related to the existence of resonance. Surprisingly, in the limit of low frequency (no resonance), the electrical averaged current exists and has always the same sign. Eventually this allows for an experimental method to measure discrete charge effect in quantum mesoscopic circuits.

Keywords: Mesoscopic circuits, quantum circuits theory, Shapiro effect, charge discreteness.

INTRODUCTION

Quantum circuits larger than atomic systems are known as mesoscopic circuits. At mesoscopic scale and low temperatures, quantum mechanics plays an important role. Particularly, some effects to considere at this scale are: heat flux quantization, charge discreteness manifestation, Casimir (electrodynamic) effect, Coulomb blockage, persistent current and others, with potential application to electronic devices, including quantum computers. Some of these effects are used in modelling sensing-devices, digital processing, and quantum computers [1]. Practical examples are found in the paper by De los Santos [2] who works in the construction of nanoelectromecanics systems (NEMS), with application to circuits in new communication trends and nano-devices. For instance, the Casimir effect could be (after him) used in tunning circuits [2] (RF Varactors). In the explicit subject of charge discreteness, persisten current, Coulomb blockade and Bloch-like oscillations could be considered in the fabrication processes of the nano-devices.

In this paper, the Shapiro effect will be studied. Shapiro effect [3] was originally outlined for a Josephson junction under a *dc* (ε_0) and *ac* ($A\cos(\omega t)$) superimposed voltages. Actually, this was the first experimental demonstration of the Josephson dynamics [4, 5, 6]. As pointed out by Feynman [6], the ac perturbation produces resonances for some characteristic frequency of the system (Josephson $\oplus$ dc). Note that Shapiro effect also has been observed in superfluid where pressure plays the role of voltage [7,8].

[1] Escuela Universitaria de Ingeniería Eléctrica - Electrónica, Universidad de Tarapacá. Casilla 6-D. Arica, Chile.
[2] Departamento de Física, Universidad de Tarapacá. Casilla 7-D. Arica, Chile.

Quantum inductive circuits with charge discreteness q_e [9-14] have a formal mathematical equivalence with Josephson junction. In fact, a quantum circuit with inductance L, applied voltage ε, and charge discreteness q_e, has the quantum Hamiltonian:

$$\hat{H} = \frac{2\hbar^2}{Lq_e}\sin^2\left(\frac{q_e}{2\hbar}\hat{\phi}\right) + \varepsilon\hat{Q} \qquad (1)$$

where the charge operator $\hat{Q}$, with discrete eigenvalues proportional to q_e, and the pseudo-flux operator $\hat{\phi}$ commute like $[\hat{Q},\hat{\phi}] = i\hbar\hat{I}$, where $\hat{I}$ is the identity operator. According to Heisenberg equation of motion $\left(\frac{d}{dt}\hat{A} = \frac{1}{i\hbar}[\hat{A},\hat{H}]\right)$, the electrical current in the circuit becomes

$$\left(\frac{d}{dt}\hat{Q} = \frac{\hbar}{Lq_e}\sin(\frac{q_e}{\hbar}\hat{\phi})\right) \qquad (2)$$

where the pseudo-flux behaves linearly in time when the emf is constant $\varepsilon = \varepsilon_0$, namely, $\frac{d}{dt}\hat{\phi} = \varepsilon_0\hat{I}$. In this way, current oscillations occur in the system with frequency $\omega_B = \frac{q_e\varepsilon_0}{\hbar}$ [13,14] with formal similarity to Bloch oscillations in Solid State Physics [15,16]. Note that the role of the lattice constant in a crystal corresponds to the elementary charge q_e in quantum circuits. For typical mesoscopic voltage $\varepsilon_0 \sim 1[Volt]$, the oscillation frequency is $\omega_B \sim 10^{15}[Hz]$ and then quite fast. So, any possible measure of oscillating electrical current is expected to be practically zero due to high fluctuations. In this paper we are concerned with measurable manifestations of charge discreteness.

In following sections, we impose an *ac* voltage on the electric quantum system. The averaged electrical current is explicitly evaluated. Next, the connection with Stark ladders is considered in a very condensed way. Conclusions are touched in the end section.

AC VOLTAGE ON THE CIRCUIT AND AVERAGED ELECTRICAL CURRENT

The formal similitude between Josephson current and equation (2) suggests to consider Shapiro effect [3, 5, 6] in quantum circuit with charge discreteness. Namely, we will consider the time depending *emf* given by $\varepsilon = \varepsilon + A\cos(\omega t)$, in the Hamiltonian (1) (see Figure 1). The evolution equation (Heisenberg) for the pseudo-flux becomes

$$\frac{d}{dt}\hat{\phi} = -\left(\varepsilon_0 + A\cos(\omega t)\right)\hat{I} \qquad (3)$$

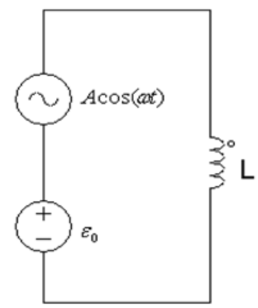

Figure 1. An electrical schematic model of the quantum circuit with inductance.

An electrical schematic model of the quantum circuit with inductance L, *dc* voltage ε_0 and *ac* voltage $A\cos(\omega t)$ is shown in Figure 1. The corresponding quantum Hamiltonian is given by (1).

For simplicity, choosing the integration constant $\phi(t=0) = 0$, the electrical current could be put explicitly as a function of time, namely,

$$\frac{d}{dt}Q = -\frac{\hbar}{Lq_e}\sin\left(\frac{q_e}{\hbar}\left(\varepsilon_0 t + \frac{A}{\omega}\sin(\omega t)\right)\right) \qquad (4)$$

Note that with the initial condition $\phi(0) = 0$ the current becomes proportional to the identity operator and then (4) is a scalar equation. If the condition of small ac voltage is assumed, the instantaneous current becomes

$$\frac{d}{dt}Q = -\frac{\hbar}{Lq_e}\left\{\sin\left(\frac{qe}{\hbar}\varepsilon_0 t\right) + \frac{q_e A}{\hbar\omega}\sin\left(\omega t\right)\cos\left(\frac{q_e}{\hbar}\varepsilon_0 t\right)\right\}$$

, when $\frac{q_e A}{\hbar\omega} << 1$ (5)

which could be written as

$$\frac{d}{dt}Q = -\frac{\hbar}{Lq_e}\sin\left(\frac{q_e}{\hbar}\varepsilon_0 t\right) - \frac{A}{2L\omega}\left(\sin\left(\omega t + \frac{q_e}{\hbar}\varepsilon_0 t\right) + \sin\left(\omega t - \frac{q_e}{\hbar}\varepsilon_0 t\right)\right) \quad (6)$$

Averaging on the Bloch period $T = \frac{h}{q_e \varepsilon_0}$, the averaged electrical current $\left\langle \frac{d}{dt}Q \right\rangle$ becomes

$$\left\langle \frac{d}{dt}Q \right\rangle = \frac{2Aq_e\varepsilon_0}{Lh}\left(\frac{\sin^2\left(\frac{\omega\hbar}{q_e\varepsilon_0}\pi\right)}{\left(\frac{q_e\varepsilon_0}{\hbar}\right)^2 - \omega^2}\right) \quad (7)$$

and then, we conclude (see figure 2):

(a) Zero averaged electric current occurs in the circuit when $\omega = n\omega_B$ where $\omega_B = \frac{q_e\varepsilon_0}{\hbar}$ and n integer.

(b) Resonances occurs when $\omega \approx (n + \frac{1}{2})\omega_B$ being the more strong when $\omega \approx \pm\frac{1}{2}\omega_B$.

(c) The more surprising fact is that for 'low frequency' $\frac{q_e A}{\hbar} < |\omega| \le \omega_B$ there is always a positive current ($\omega \neq 0$). In fact, for small frequency we have $\left\langle \frac{d}{dt}Q \right\rangle = \frac{A\hbar^3\pi}{Lq_e^3\varepsilon_0^3}\omega^2$.

Figure 2 shows the averaged current as a function of the external frequency ω. Note that for $|\omega\hbar / q_e\varepsilon_0| < 1$ the current is always positive ($\omega \neq 0$).

This is a physical effect detectable by direct current measurements.

Averaged Current (Arbitrary Units)

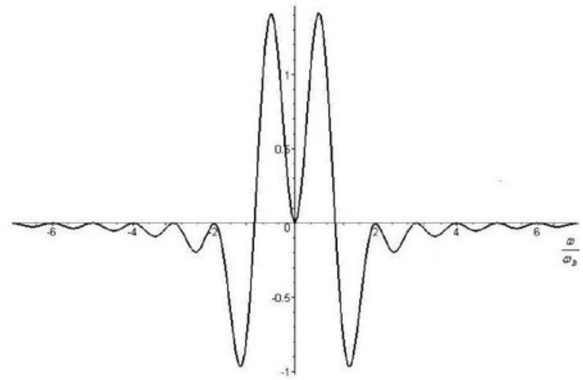

Figure 2. The average current as function of the frecuency ω.

STARK LADDERS

As pointed out in reference [14], oscillations similar to the one described in the introduction, with $\omega_B = \frac{q_e\varepsilon_0}{\hbar}$ are consequence of the discrete energy spectrum of the systems (Stark ladders). These results are shown briefly as follows: the Schrödinger equation for a *dc* voltage becomes directly from the Hamiltonian (1). In the pseudo-flux representation we have:

$$\left\{\frac{2\hbar^2}{Lq_e^2}\sin^2\left(\frac{q_e}{2\hbar}\phi\right)\right\}\psi(\phi) + \varepsilon_0 i\hbar\frac{\partial}{\partial\phi}\psi(\phi) = E\psi(\phi) \quad (8)$$

The direct solution is given by

$$\psi(\phi) = \psi_0 \exp\left\{-\frac{i}{\hbar\varepsilon_0}\left(\left(E - \frac{\hbar^2}{Lq_e^2}\right)\phi + \frac{\hbar^3}{Lq_e^3}\sin\frac{q_e}{\hbar}\phi\right)\right\} \quad (9)$$

The condition of charge discreteness ensures that the wavefunction in pseudo-flux space must be periodic. Namely, $\psi(\phi) = \psi(\phi + \frac{h}{q_e})$ and then we have the condition of discreteness on the spectrum:

$$E_n = nq_e\varepsilon_0 + \frac{\hbar^2}{Lq_e^2},\ (n \in Z) \quad (10)$$

as conjectured in Ref. [14] for oscillations in electrical current and Stark ladder.

CONCLUSIONS

The averaged electrical current (7) of the quantum inductive circuit, with charge discreteness and superimposed *ac* and *dc* voltages (Figure 1), was studied. The electrical averaged current is zero for external frequencies where $\omega = n\omega_B = n\frac{q_e \varepsilon_0}{\hbar}$.

Resonances occur when $\omega \approx (n+\frac{1}{2})\omega_B$.

The most important point: for low *ac* frequency ω, the system has a non zero averaged electrical current.

Stark ladders were obtained in a compact analytical way, as described in [14].

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