

## CHIRAL WAVES IN A METAMATERIAL MEDIUM

## ONDAS QUIRALES EN UN MEDIO METAMATERIAL

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### RESUMEN

En este trabajo se estudia la refracción anómala en el borde de un medio metamaterial con fuerte quiralidad. El hecho de que para una onda monocromática el vector de Poynting es antiparalelo a la dirección de la velocidad de fase conduce a relevantes propiedades que pueden tener ventajas en el diseño de novedosos dispositivos y componentes a frecuencias de microondas.

Palabras clave: Ondas quirales, metamateriales.

### ABSTRACT

*In this paper we study the anomalous refraction at the boundary of a metamaterial medium with strong chirality. The fact that for a time-harmonic monochromatic plane wave the direction of the Poynting vector is antiparallel with the direction of phase velocity, leads to exciting features that can be advantageous in the design of novel devices and components at microwaves frequencies.*

*Keywords: Chiral waves, metamaterial.*

### INTRODUCTION

Composite materials in which both permittivity and permeability possess negative values at some frequencies has recently gained considerable attention. This idea was originally initiated by Veselago in 1967, who theoretically studied plane wave propagation in a material whose permittivity and permeability were assumed to be simultaneously negative. Recently have been constructed such a composite medium for the microwave regime, and experimentally the presence of anomalous refraction in this medium is verified [1]. Previous theoretical study of electromagnetic wave interaction with omega media using the circuit-model approach had also revealed the possibility of having negative permittivity and permeability in omega media for certain range of frequencies [2]. That is important for design of circularly polarized antennas.

The anomalous refraction at the boundary of such a medium with a conventional medium, and the fact that for a time-harmonic monochromatic plane wave the direction

of the Poynting vector is antiparallel with the direction of phase velocity, can lead to exciting features that can be advantageous in design of novel devices and components. For instance, as a potential application of this material, the idea of compact cavity resonators in which a combination of a slab of conventional material and a slab of metamaterial with negative permittivity and permeability. The problems of radiation, scattering, and guidance of electromagnetic waves in metamaterials with negative permittivity and permeability, and in media in which the combined paired layers of such media together with the conventional media are present, can possess very interesting features leading to various ideas for future potential applications such as phase conjugators, unconventional guided-wave structures, compact thin cavities, thin absorbing layers, high-impedance surfaces, to name a few. In this talk, we will first present a brief overview of electromagnetic properties of the media with negative permittivity and permeability, and we will then discuss some ideas for potential applications of these materials. In this work we discuss the chiral waves in metamaterial media.

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## CHIRAL WAVES

In classical electrodynamics, the response (typically frequency dependent) of a material to electric and magnetic fields is characterized by two fundamental quantities, the permittivity  $\epsilon$  and the permeability  $\mu$ . The permittivity relates the electric displacement field  $\vec{D}$  to the electric field  $\vec{E}$  through  $\vec{D} = \epsilon\vec{E}$ , and the permeability  $\mu$  relates the magnetic field  $\vec{B}$  and  $\vec{H}$  by  $\vec{B} = \mu\vec{H}$ . If we do not take losses into account and treat  $\epsilon$  and  $\mu$  as real numbers, according to Maxwell's equations, electromagnetic waves can propagate through a material only if the index of refraction  $n$ , is real. Dissipation will add imaginary components to  $\epsilon$  and  $\mu$  cause losses, but for a qualitative picture, one can ignore losses and treat  $\epsilon$  and  $\mu$  as real numbers. Also, strictly speaking,  $\epsilon$  and  $\mu$  are second-rank tensors, but they reduce to scalars for isotropic materials. In a medium with  $\epsilon$  and  $\mu$  both positive, the index of refraction is real and electromagnetic waves can propagate. All our everyday transparent materials are such kind of media. In a medium where one of the  $\epsilon$  and  $\mu$  is negative but the other is positive, the index of refraction is imaginary and electromagnetic waves cannot propagate. Metals and Earth's ionosphere are such kind of media. In fact, the electromagnetic response of metals in the visible and near-ultraviolet regions is dominated by the negative epsilon concept [1-3].

Although all our everyday transparent materials have both positive  $\epsilon$  and positive  $\mu$ , from the theoretical point of view, in a medium with  $\epsilon$  and  $\mu$  both negative, electromagnetic waves can also propagate through. Moreover, if such media exist, the propagation of waves through them should give rise to several peculiar properties. This was first pointed out by Veselago over 30 years ago when no material with simultaneously negative  $\epsilon$  and  $\mu$  was known [4]. For example, the cross product of  $\vec{E}$  and  $\vec{H}$  for a plane wave in regular media gives the direction of both propagation and energy flow, and the electric field  $\vec{E}$ , the magnetic field  $\vec{H}$ , and the wave vector  $\vec{k}$  form a right-handed triplet of vectors. In contrast, in a medium with  $\epsilon$  and  $\mu$  both negative,  $\vec{E} \times \vec{H}$  for a plane wave still gives the direction of energy flow, but the wave itself that is, the phase velocity propagates in the opposite direction, i.e., wave vector  $\vec{k}$  lies in the opposite direction of  $\vec{E} \times \vec{H}$  for propagating waves. In this case, electric field  $\vec{E}$ , magnetic field  $\vec{H}$ , and wave vector  $\vec{k}$  form a left-handed triplet of vectors.

Such a medium is therefore termed left-handed medium [5]. In addition to this "left-handed" characteristic, there

are a number of other dramatically different propagation characteristics stemming from a simultaneous change of the signs of  $\epsilon$  and  $\mu$ , including reversal of both the Doppler shift and the Cerenkov radiation, anomalous refraction, and even reversal of radiation pressure to radiation tension. However, although these counterintuitive properties follow directly from Maxwell's equations, which still hold in these unusual materials. Such type of left-handed materials have never been found in nature but such media can be prepared artificially, they will offer exciting opportunities to explore new physics and potential applications in the area of radiation-material interactions. Following the suggestion of Pendry, Smith and co-workers reported that a medium made up of an array of conducting nonmagnetic split ring resonators and continuous thin wires can have both an effective negative permittivity  $\epsilon$  and negative permeability  $\mu$  for electromagnetic waves propagating in some special direction and special polarization at microwave frequencies [5]. This is the first experimental realization of an artificial preparation of a left-handed material, where on the one hand, the permittivity of metallic particles is automatically negative at frequencies less than the plasma frequency, and on the other hand, the effective permeability of ferromagnetic materials for electromagnetic waves propagating in some particular direction and polarization can be negative at frequency in the vicinity of the ferromagnetic resonance frequency, which is usually in the frequency region of microwaves. However this configuration exhibit chirality and a rotation of the polarization so the analysis of metamaterial presented by several authors provides a good but not exact characterization of the metamaterial [6]. The evidence of chirality behavior suggests that if it is included in the conditions to obtain a metamaterial behavior of a medium further progress will be obtained. In this short paper, we propose to investigate the conditions to obtain a metamaterials having simultaneously negative  $\epsilon$  and negative  $\mu$  and very low eddy current loss. As a initial point, we consider a media where the electric polarization  $\vec{P}$  depends not only on the electric field  $\vec{E}$ , and the magnetization  $\vec{M}$  depends not only on the magnetic field  $\vec{H}$ , and we may have, for example, constitutive relations given by the Born-Federov formalism [7].

$$\vec{D}(\vec{r}, \omega) = \epsilon(\omega)(\vec{E}(\vec{r}, \omega) + T(\omega)\nabla \times \vec{E}(\vec{r}, \omega)) \quad (1)$$

$$\vec{B}(\vec{r}, \omega) = \mu(\omega)(\vec{H}(\vec{r}, \omega) + T(\omega)\nabla \times \vec{H}(\vec{r}, \omega)) \quad (2)$$

The pseudoscalar  $T$  represents the chirality of the material and it has length units [7]. In the limit  $T \rightarrow 0$ , the constitutive relations (1) and (2) for a standard linear isotropic lossless dielectric with permittivity  $\varepsilon$  and permeability  $\mu$  are recovered.

According to Maxwell's equations, electromagnetic waves propagating in the direction of magnetization in a homogeneous magnetic material is either positive or negative transverse circularly polarized. If the composite can truly be treated as a homogeneous magnetic system in the case of grain sizes much smaller than the characteristic wavelength, electric and magnetic fields in the composite should also be either positive or negative circularly polarized and can be expressed as

$$\vec{E}^{\pm}(\vec{r}, t) = \hat{E}_0^{(\pm)} e^{-j(k_{\pm}z - \omega_0 t)} \quad (3)$$

$$\vec{H}^{\pm}(\vec{r}, t) = \hat{H}_0^{(\pm)} e^{-j(k_{\pm}z - \omega_0 t)} \quad (4)$$

where  $E_0^{\pm} = E_0(\hat{x} \pm \hat{y})$ , and  $\nabla \times \vec{E}^{\pm}(\vec{r}, t) = \mp k_{\pm} \vec{E}^{\pm}$ ,  $k_{\pm} \geq 0$  is the chiral wave number.

In this case of right polarized wave we can see that the effective permittivity  $\varepsilon_p$  and the effective permeability  $\mu_p$  are obtained from

$$\begin{aligned} \int \vec{D}(\vec{r}, \omega) e^{-jk_+z} d\vec{r} &= \varepsilon \int (\vec{E}(\vec{r}, \omega) + T \nabla_x \vec{E}) e^{-jk_+z} d\vec{r} \\ &= \varepsilon \int (1 - k_+ T) \vec{E} e^{-jk_+z} d\vec{r} \end{aligned} \quad (5)$$

with  $\varepsilon_p = \varepsilon(1 - k_+ T)$  and  $k_+ T \geq 1$ .

Similarly, we have

$$\begin{aligned} \int \vec{B}(\vec{r}, \omega) e^{-jk_+z} d\vec{r} &= \mu_{eff} \int \vec{H}(\vec{r}, \omega) e^{-jk_+z} d\vec{r} \\ &= \mu(1 - k_+ T) \int \vec{H}(\vec{r}, \omega) e^{-jk_+z} d\vec{r} \end{aligned} \quad (6)$$

with  $\mu_p = \mu(1 - k_+ T)$  and where  $k_{eff}$  and  $\omega$  are related by  $k_{eff}^2 = \omega^2(\varepsilon_p \mu_p)$ . Equations (5) and (6) are exact in principle assuming that nonlocal effects can be neglected. This assumption is appropriate in many cases. But in some cases, nonlocal effects can be significant and cannot be neglected, as has been shown in the past. In such cases, Eqs. (5) and (6) shall be not exact. For simplicity, in this paper we have assumed that nonlocal effects can be neglected and hence Eqs. (5) and (6) shall be valid. In Eqs. (3 and (4) the sign of the effective wave number can be positive or negative depending on the product  $k_+ T$  and the energy flow. For convenience we assume that the direction of energy flow is in the positive direction of the  $z$  axis, but the sign of  $k_{eff}$  still can be positive or negative. In the case of right polarization, if  $1 \geq k_+ T \geq 0$ , the phase velocity and energy flow are in the same directions, and from Maxwell's equation, one can see that the electric  $\vec{E}$  and magnetic field  $\vec{H}$  and the wave vector  $\vec{k}_{eff}$  will form a right-handed triplet of vectors. This is the usual case for right-handed materials. In contrast, if  $k_+ T \geq 1$  the phase velocity and energy flow are in opposite directions, and  $\vec{E}$ ,  $\vec{H}$ , and  $\vec{k}_{eff}$  will form a left-handed triplet of vectors. This is just the peculiar case for left handed materials where the effective permittivity  $\varepsilon_{eff}$  and the effective permeability  $\mu_{eff}$  are simultaneously negative. So, for incident waves of a given frequency  $\nu$ , we can determine whether wave propagation in the composite is right handed or left handed through the relative sign changes of  $k_{eff}$ .

Based on Eqs. (5)-(6), we have computed  $T$  and  $\varepsilon/\varepsilon_p$  or  $\mu/\mu_p$  versus  $\kappa / \sqrt{\mu_p \varepsilon_p}$ , as shown in Fig. 1 When  $\kappa$  is very close to  $\sqrt{\mu_p \varepsilon_p}$ , the value of  $T$  is quite large, indicating a strong spatial dispersion. Hence the singular point is the very point of traditional limitation. However, with  $\kappa / \sqrt{\mu_p \varepsilon_p}$  continuously increasing, the spatial dispersion strength falls down very quickly. Therefore, if  $\kappa$  is not around  $\sqrt{\mu_p \varepsilon_p}$ , e.g.  $\kappa < 0.7\sqrt{\mu_p \varepsilon_p}$  or  $\kappa > 1.3\sqrt{\mu_p \varepsilon_p}$ , we need not take nonlinear terms into consideration at all. Hence the strong spatial dispersion and nonlinearity cannot put the upper limitation to chirality parameters either.

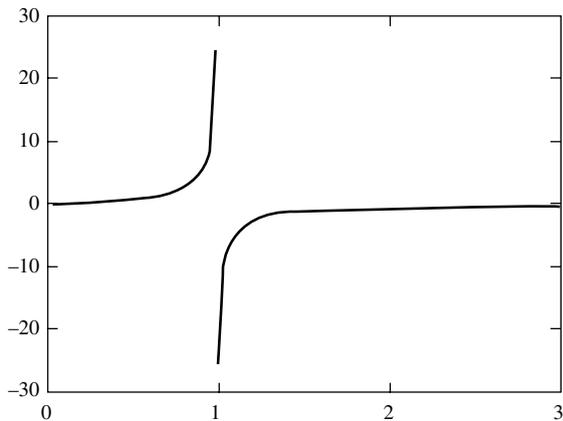


Figure 1. The strength relationship of chirality and spatial dispersion.  $T$  versus  $\kappa / \sqrt{\mu_p \epsilon_p}$ . The point of  $\kappa / \sqrt{\mu_p \epsilon_p} = 1$  is singularity, corresponding infinite spatial dispersion coefficient  $T$ . When  $\kappa / \sqrt{\mu_p \epsilon_p} > 1$ ,  $T$  becomes negative for keeping the positive rotation term coefficients with negative  $\mu$  and  $\epsilon$ .

### CONCLUSIONS

From figure 1, it is clear that enhancing spatial dispersion will not lead to strong chirality and will reach the traditional limitation point. This is why we have never succeeded in realizing strong chirality no matter how to improve the asymmetry and spatial dispersion.

Fortunately, as pointed out earlier, the strong chirality does not require strong spatial dispersion. Hence the most important difference between strong and weak chirality is that  $T$  and  $\kappa$  have opposite signs, which necessarily leads to negative  $\epsilon$  and  $\mu$ . Here,  $\kappa$  stands for chirality and  $T$  is the chiral coefficient of the first order for spatial dispersion. Strong chirality roots from using one type of spatial dispersion to get the conjugate stereoisomer, or chirality. It is an essential condition for supporting the backward eigenwave in strong chiral medium.

In conclusion, a strong chiral medium behaves like Veselago's medium. Under the weak spatial dispersion, the energy is always positive for chiral medium. We show that strong chirality does not equal strong spatial dispersion, which occurs only around a singular point. Even in this small region with very strong spatial

dispersion, the Pasteur model is meaningful. Neither spatial dispersion nor energy will hinder chirality to be stronger, but we cannot realize strong chirality only by increasing the spatial dispersion. The necessary condition of strong chiral medium is that the chirality and spatial dispersion are of conjugated types. We remark that strong chiral media have found wide applications in the negative refraction and supporting of backward waves, useful in metamaterial substrates.

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